CASIO CLASSWIZ Non-programmable Scientific Calculator

Matrices

June 2004 P2 Q11(a)

11 **(a)** In a swimming match between two schools, *C* and *D*, two students from each school took part in each event.
The number of places each school gained in each position is shown in the table.

	First	Second	Third	Fourth
School *C*	6	3	5	6
School *D*	4	7	5	4

The points awarded for First, Second, Third and Fourth places were 5, 3, 1 and 0 respectively.

Matrices related to this information are defined below.

$$\mathbf{A} = \begin{pmatrix} 6 & 3 & 5 & 6 \\ 4 & 7 & 5 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 5 \\ 3 \\ 1 \\ 0 \end{pmatrix}$$

(i) What does the sum of the elements in each column of **A** represent? [1]

(ii) **(a)** Find **AB**. [2]

(b) What information is shown by **AB**? [1]

(iii) It was suggested that the points awarded for First, Second, Third and Fourth places should have been 5, 3, 2 and 1 respectively.

Would this suggestion have made any difference to which school won this match?
Show clear working to justify your answer. [1]

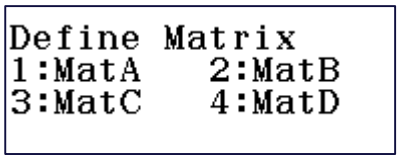
Solution:

(i) It shows the number of events.

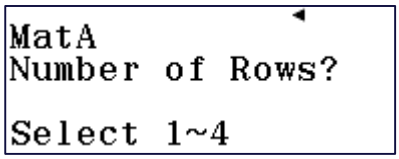
(ii) (a) First we need to define Matrix A and B in the memory.

To do this press MENU 4

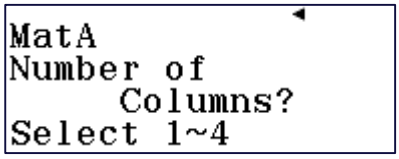
and define the matrices A and B as shown below.



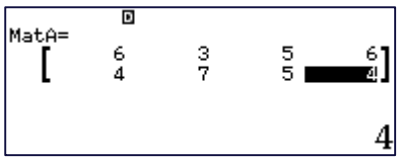
Enter no of rows for Matrix A i.e. 2



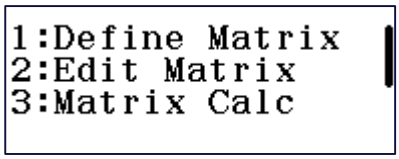
Enter no. of columns for Matrix A i.e. 4

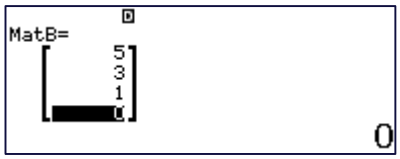


Now type in the values of corresponding elements according to question

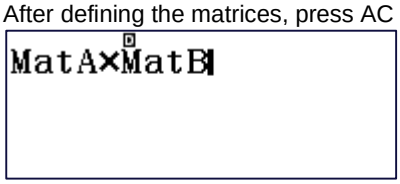


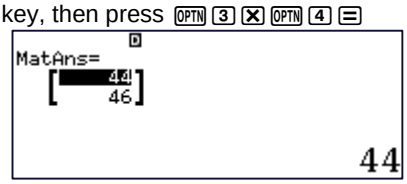
Once we have defined Matrix A, we will define Matrix B by pressing [OPTN] Key and selecting menu 1 i.e. Define Matrix.





After defining the matrices, press AC key, then press [OPTN] [3] [×] [OPTN] [4] [=]





(b) It shows total points earned by each school. i.e. 44 by school C and 46 by school D.

(iii) Yes it would, and the result will be $\binom{55}{55}$ which will be a tie between the two.

June 2006 P2 Q11(a)

11 (a) $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 3 & -2 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -2p & 3p \\ -3p & p \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

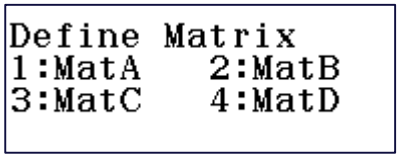
(i) Evaluate $4\mathbf{C} - 2\mathbf{A}$. [2]

(ii) Given that $\mathbf{B} = \mathbf{A}^{-1}$, find the value of p. [2]

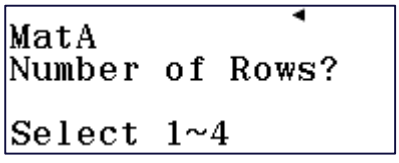
(iii) Find the 2×2 matrix $\mathbf{X}$, where $\mathbf{AX} = \mathbf{C}$. [2]

Solution:

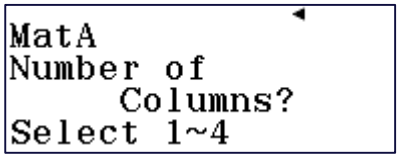
First we need to define Matrix A and C in the memory. To do this press [MENU] [4] and define the matrices A and C as shown below.



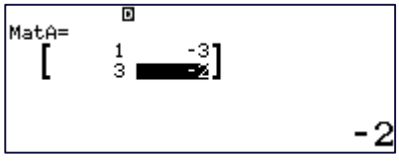
Enter no of rows for Matrix A i.e. 2



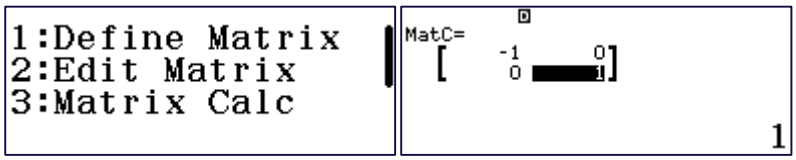
Enter no. of columns for Matrix C i.e. 2



Now type in the values of corresponding elements according to question



Once we have defined Matrix A, we will define Matrix C by pressing [OPTN] Key and selecting menu 1 i.e. Define Matrix.

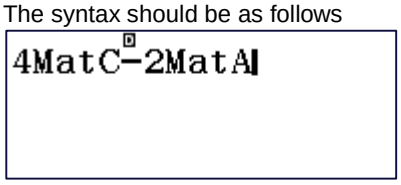


(i) **4C – 2A**

Once we have defined the matrices A and C. It is very easy to perform the indicated operation in question. After defining matrices and then press [AC]

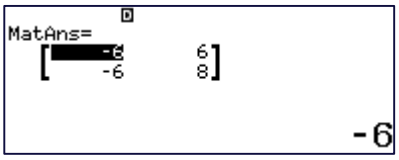
Now type [4] then [OPTN] [5] [−] [2] [OPTN] [3]

The syntax should be as follows



Now press [=]

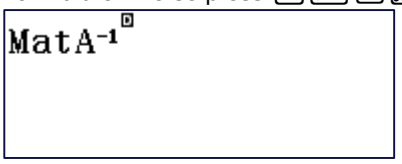
The desired answer is on the screen

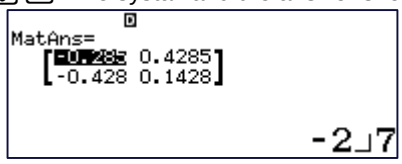


(ii) **Find p if $B=A^{-1}$**

First of all we need to find inverse of A.

To find the inverse press [AC] [OPTN] [3] [x^{-1}] [=] The systax and the answer should be as follows.





Note: Navigate with the help of arrow direction (arrow) keys to show decimal in equivalent fraction forms.

It can be clearly seen by comparing B and A^{-1} that $p = \frac{1}{7}$

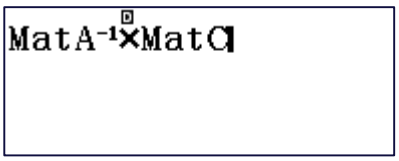
(iii) **Finding X if AX=C**

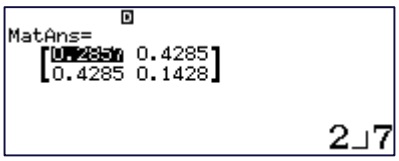
AX = C

⇨ $X = A^{-1}C$

Since we have already found A^{-1} that answer is to be multiplied by C.

To do this go to matrix menu and type [OPTN] [3] [x^{-1}] [=] The systax and the answer should be as follows.





Note: Navigate with the help of arrow direction (arrow) keys to show decimal in equivalent fraction forms.

June 2007 P2 Q11(c)

(c) $\mathbf{M} = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$

(i) Find the determinant of **M**. [1]

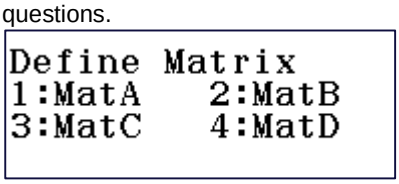
(ii) Write down the inverse of **M**. [1]

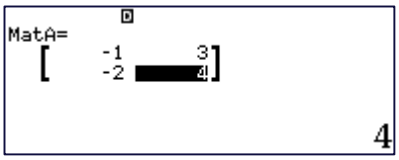
(iii) Find the matrix **X**, where $\mathbf{MX} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$. [2]

Solution:

First we define Matrix M (let it be A).

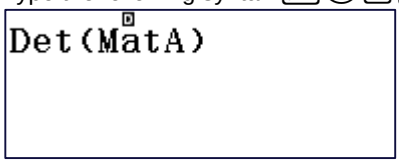
To do this press MENU 4 and define the matrix A. write the number of rows and column as done in previous questions.

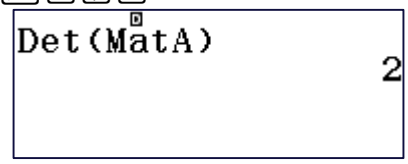




(i) **Determinant of M**

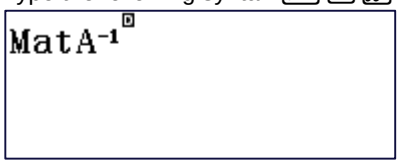
Type the following syntax OPTN ▼ 2 OPTN 3) =

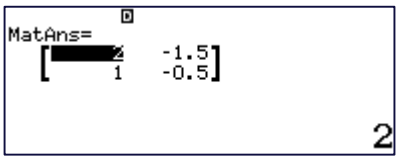




(ii) **Inverse of M**

Type the following syntax OPTN 3 x^{-1} =





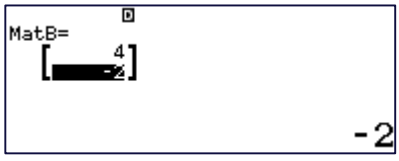
(iii) **Finding X if MX is given.**

Since MX = $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

=> X = $M^{-1} \times \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

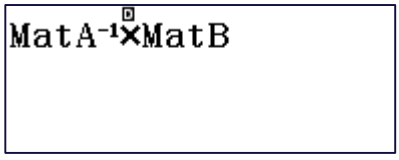
Define $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ as Matrix B in the calculator.

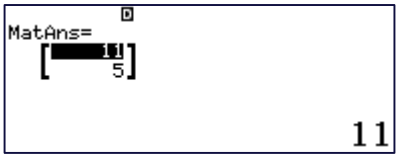
To do this, pressing OPTN Key and select menu 1 i.e. Define Matrix and let it be B.



Once we have done defining matrix B, we can obtain the answer by multiplying A^{-1} by B.

To do this press OPTN 3 x^{-1} × OPTN 4 =





so final answer is $x = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$

CASIO CLASSWIZ Non-programmable Scientific Calculator

June 2008 P2 Q5 (c)

(c) Ann went on a car journey that was split into three stages.
Two relevant matrices are shown below.
The first matrix shows the average speed, in kilometres per hour, of the car during each stage.
The second matrix shows the time, in hours, taken for each stage.

$$\begin{array}{c} \\ \text{Average speed} \end{array} \begin{array}{c} \begin{array}{ccc} \text{First stage} & \text{Second stage} & \text{Third stage} \end{array} \\ \begin{pmatrix} 40 & 30 & 50 \end{pmatrix} \end{array} \qquad \begin{array}{c} \text{Time} \\ \begin{pmatrix} 1\frac{1}{2} \\ 1 \\ 2\frac{1}{2} \end{pmatrix} \end{array} \begin{array}{l} \\ \text{First stage} \\ \text{Second stage} \\ \text{Third stage} \end{array}$$

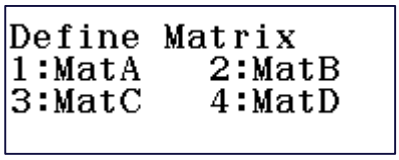
(i) Find $\begin{pmatrix} 40 & 30 & 50 \end{pmatrix} \begin{pmatrix} 1\frac{1}{2} \\ 1 \\ 2\frac{1}{2} \end{pmatrix}$. [1]

(ii) What information is given by the matrix obtained in part **(i)**? [1]

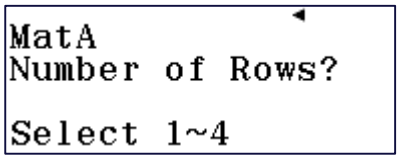
(iii) Calculate the average speed for the whole journey. [1]

Solution:

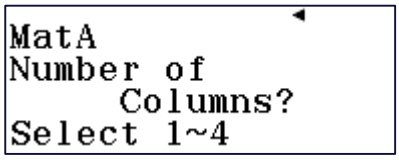
(i) Suppose the two matrices are A and B respectively,
First we need to define Matrix A and B in the memory.
To do this press MENU 4
and define the matrices A and B as shown below.



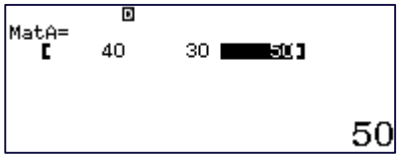
Enter no of rows for Matrix A i.e. 1



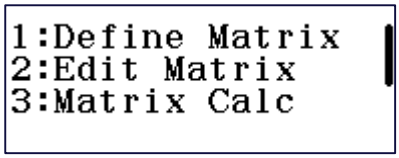
Enter no. of columns for Matrix A i.e. 3

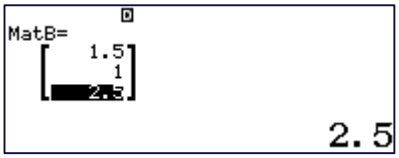


Now type in the values of corresponding elements according to question

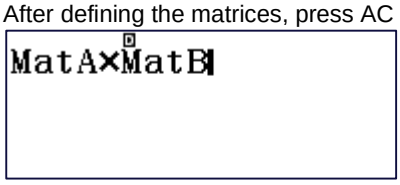


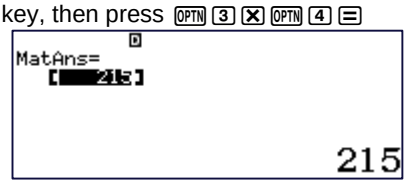
Once we have defined Matrix A, we will define Matrix B by pressing OPTN Key and selecting menu 1 i.e. Define Matrix.





we must enter the values of mixed fraction in improper fraction form.
After defining the matrices, press AC key, then press OPTN 3 × OPTN 4 =





(ii) Total distance for the whole journey as speed × time = distance travelled.

(iii) Average speed = $\frac{total\ distance}{total\ time} = \frac{215}{1.5+1+2.5} = \frac{215}{5} = 43\ km/h$

November 2009 P2 Q5

5 **(a)** Evaluate

(i) $3\begin{pmatrix}2\\4\\0\end{pmatrix} - 2\begin{pmatrix}1\\6\\-3\end{pmatrix}$, [2]

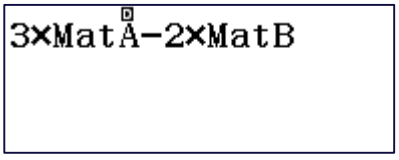
(ii) $(1\ \ 3\ \ 4)\begin{pmatrix}0 & 4\\3 & 1\\5 & 0\end{pmatrix}$. [2]

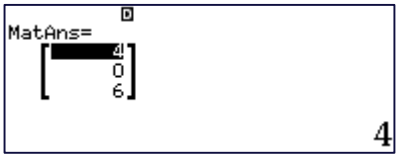
(b) $\mathbf{A} = \begin{pmatrix}2 & -3\\0 & 1\end{pmatrix}$

(i) Find $\mathbf{A}^{-1}$. [2]

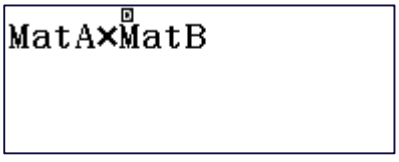
Solution:

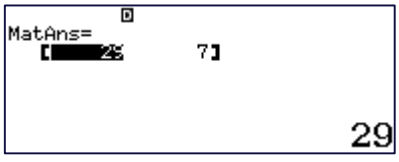
(i) Suppose the two matrices are A and B respectively,
First we need to define Matrix A and B in the memory in the same way that we have been doing in previous sums. Once we have defined the matrices, we will enter the following syntax in the calculator. 3 × OPTN 3 − 2 × OPTN 4 =
The screen will be as follows.



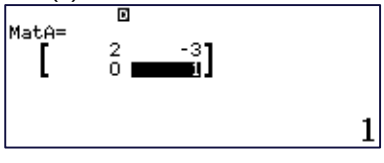


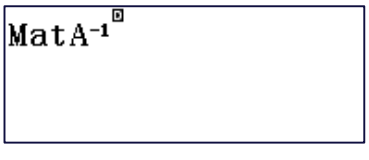
(ii) Suppose the two matrices are A and B respectively,
First we need to define Matrix A and B in the memory in the same way that we have been doing in previous sums. Once we have defined the matrices, we will enter the following syntax in the calculator. OPTN 3 × OPTN 4 =
The screen will be as follows.

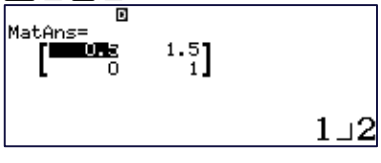




(b) First we will define matrix A and then type the following syntax OPTN 3 x^{-1} =







June 2010 P2 Q5 (a), (b)

5 **(a)** Bertie goes shopping and buys three different types of fruit.
The first matrix below shows the number of kilograms of each fruit bought during two different weeks.
The second matrix shows the price per kilogram, in cents, of each fruit.

$$\begin{array}{l} \\ \text{Week 1} \\ \text{Week 2} \end{array} \begin{array}{c} \begin{array}{ccc} \text{bananas} & \text{apples} & \text{grapes} \end{array} \\ \begin{pmatrix} 1 & 2 & 0.5 \\ 1.5 & 1 & 1 \end{pmatrix} \end{array} \qquad \begin{array}{c} \text{price/kg} \\ \begin{pmatrix} 290 \\ 160 \\ 640 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{bananas} \\ \text{apples} \\ \text{grapes} \end{array}$$

(i) $\mathbf{F} = \begin{pmatrix} 1 & 2 & 0.5 \\ 1.5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 290 \\ 160 \\ 640 \end{pmatrix}$.

Find **F**. [2]

(ii) Explain the meaning of the information given by the matrix **F**. [1]

(iii) Find the total amount of money, in dollars, that Bertie spent on fruit during the two weeks. [1]

(b) The matrix **M** satisfies the equation

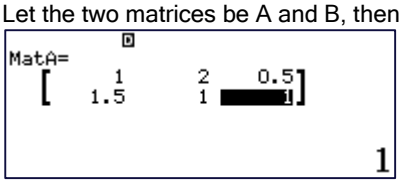
$$8\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} + 5\mathbf{M} = \mathbf{M}.$$

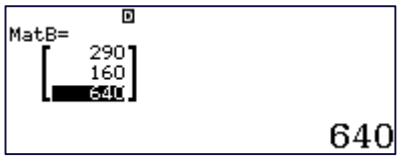
Find **M**. [2]

Solution:

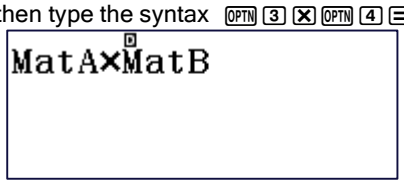
(a)

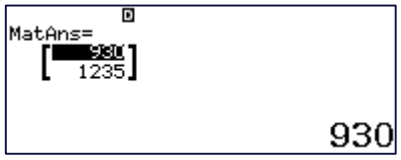
(i) Let the two matrices be A and B, then $F = A \times B$. Define the matrices A and B in calculator





then type the syntax OPTN 3 × OPTN 4 =





(ii) The amount of money spent on fruits by Bertie during the two weeks

(iii) \$9.30 + \$12.35 = \$21.65

(b) $8\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} + 5M = M$

$8\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} = M - 5M$

$8\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} = -4M$

$-2\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} = M$

$M = \begin{pmatrix} -6 & 0 \\ 2 & -4 \end{pmatrix}$

June 2011 P2 Q8 (a)

8 (a) $A = \begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix}$ *and* $B = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$

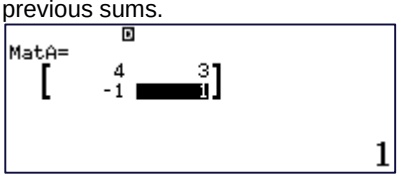
Find

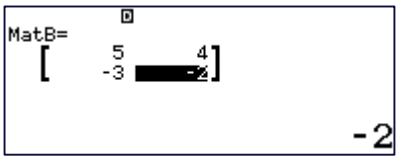
(i) 2A – B [2]

(ii) A^{-1} [2]

Solution:

(i) First we need to define Matrix A and B in the memory in the same way that we have been doing in previous sums.

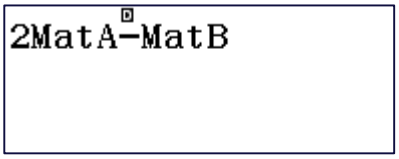


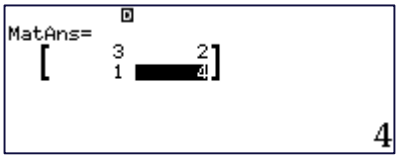


Once we have defined the matrices, we will enter the following syntax in the calculator.

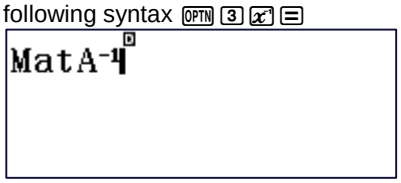
2 OPTN 3 − OPTN 4 =

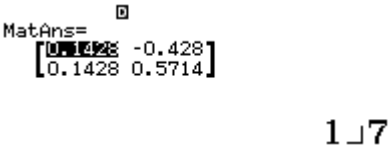
The screen will be as follows.





(ii) Since we have already defined the matrices, we just need to find inverse of matrix A by the following syntax OPTN 3 x^{-1} =





1⌟7

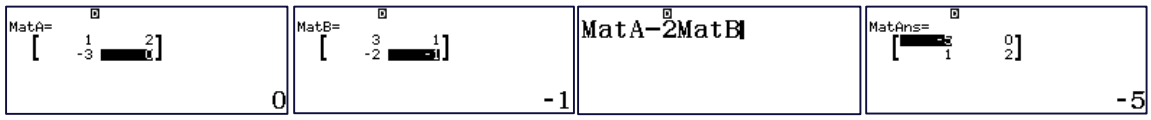
CASIO CLASSWIZ Non-programmable Scientific Calculator

June 2012 P2 Q9 (a), (b)

9 (a) $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix}$

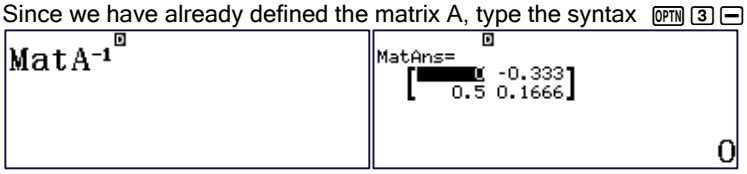
(i) Find $\mathbf{A} - 2\mathbf{B}$.

Define the two matrices be A and B in calculator then type the syntax OPTN 3 − 2 OPTN 4 =



(ii) Find $\mathbf{A}^{-1}$.

Since we have already defined the matrix A, type the syntax OPTN 3 − 2 OPTN 4 =



(b) Zara is going to put carpet and underlay in three rooms, A, B and C, of her house.
The cost per square metre for the carpet in A is \$18, in B is \$22 and in C is \$25.
The cost per square metre for the underlay is \$6 in A and \$8 in the other two rooms.
This information is represented by matrix **P** below.

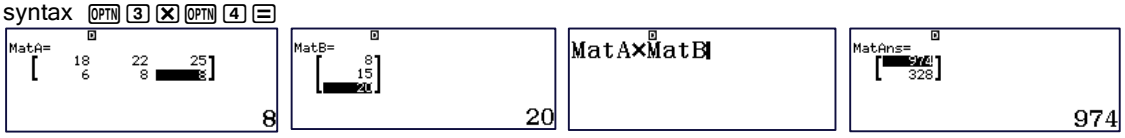
$$\mathbf{P} = \begin{pmatrix} 18 & 22 & 25 \\ 6 & 8 & 8 \end{pmatrix}$$

The amount of carpet and underlay required for A, B and C is $8\,m^2$, $15\,m^2$ and $20\,m^2$ respectively.
This information is represented by matrix **Q** below.

$$\mathbf{Q} = \begin{pmatrix} 8 \\ 15 \\ 20 \end{pmatrix}$$

(i) Find **PQ**.

Let the two matrices be A and B, then $PQ = A \times B$. Define the matrices A and B in calculator then type the syntax OPTN 3 × OPTN 4 =



Answer [2]

(ii) Explain what the matrix **PQ** represents.

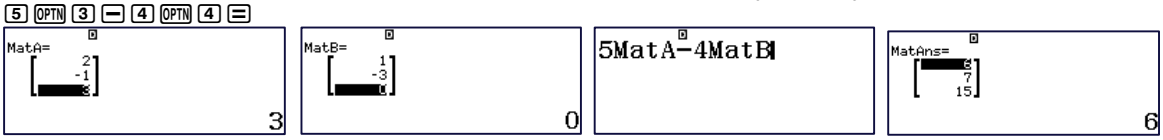
Answer It represents the costs of carpet and overlay for all three rooms are 974 and 328 respectively [1]

November 2013 P2 Q7

7 (a) Express as a single matrix $5\begin{pmatrix}2\\-1\\3\end{pmatrix}-4\begin{pmatrix}1\\-3\\0\end{pmatrix}$.

Let the two matrices be A and B, define the matrices in calculator then type the syntax

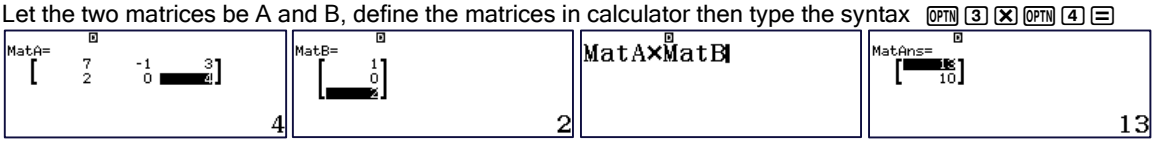
5 OPTN 3 − 4 OPTN 4 =



Answer [2]

(b) Express as a single matrix $\begin{pmatrix}7&-1&3\\2&0&4\end{pmatrix}\begin{pmatrix}1\\0\\2\end{pmatrix}$.

Let the two matrices be A and B, define the matrices in calculator then type the syntax OPTN 3 × OPTN 4 =

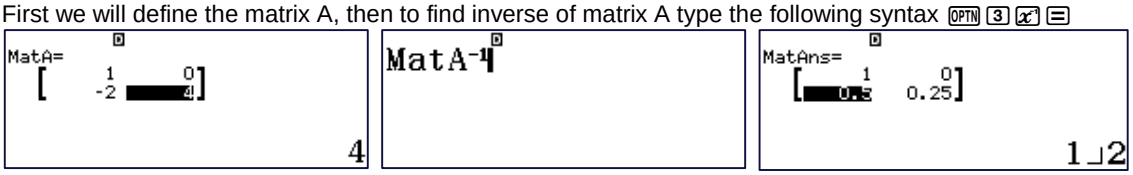


Answer [2]

(c) $\mathbf{A}=\begin{pmatrix}1&0\\-2&4\end{pmatrix}$

(i) Find $\mathbf{A}^{-1}$.

First we will define the matrix A, then to find inverse of matrix A type the following syntax OPTN 3 x^{-1} =



Answer $\begin{pmatrix}&\\&\end{pmatrix}$ [2]

(ii) $\mathbf{B}+3\mathbf{I}=\mathbf{A}$ where $\mathbf{I}$ is the 2×2 identity matrix.

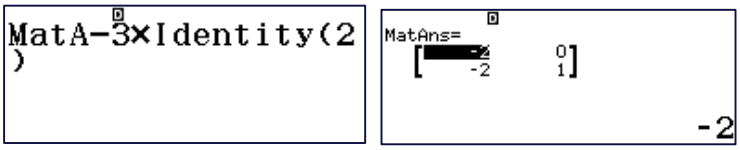
Find $\mathbf{B}$.

B+3I=A

⇨ B = A – 3I

Since we have already defined matrix A, we need to enter the syntax as follows

OPTN 3 − 3 × OPTN ▼ 4 2) =



Answer $\begin{pmatrix}&\\&\end{pmatrix}$ [2]