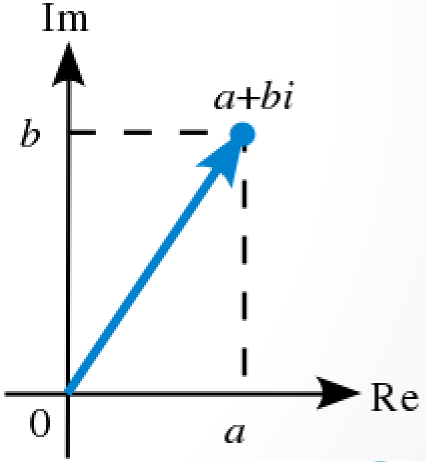
Chapter 1 Number System

Video by Umair Sajid
Phone number XXXXXXXX
Youtube channel math and math

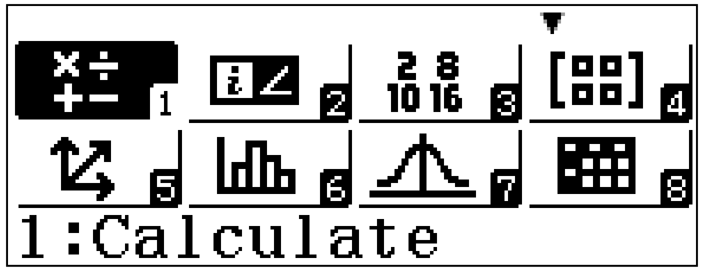
Complex number

- A **complex number** is a quantity of the form x + iy, where x and y are real **numbers**, and "i" represents the unit imaginary numbers equal to the positive square root of -1.

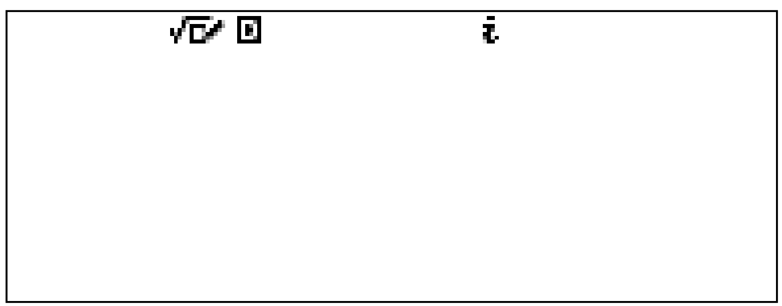
$$i = \sqrt{-1}$$
$$\Rightarrow i^2 = -1$$



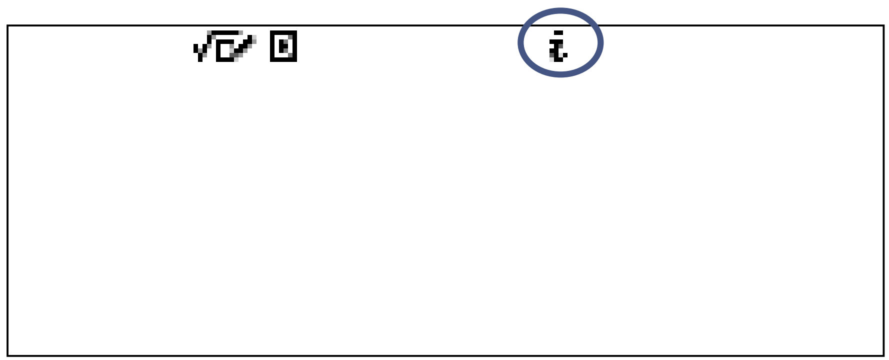
Press menu



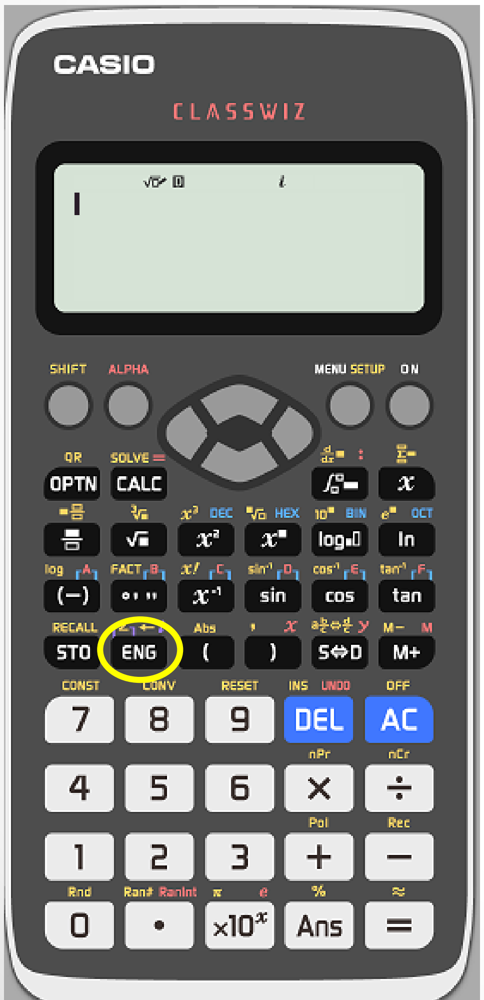
Press 2

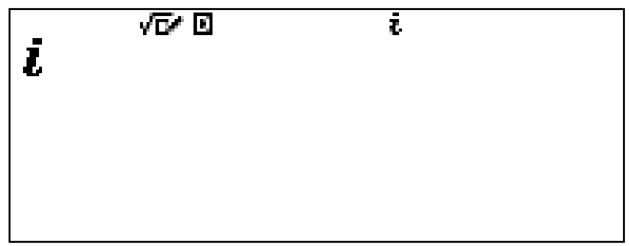


your calculator screen look like this



For i symbol press ENG





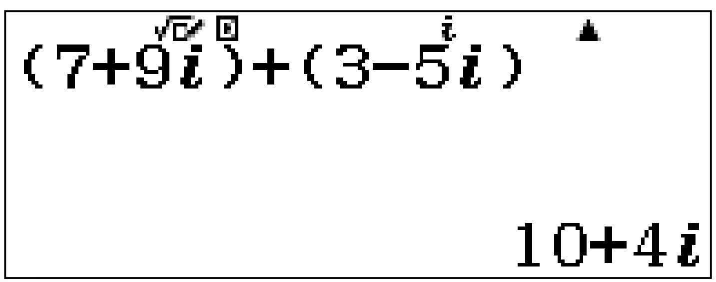
Addition of complex numbers

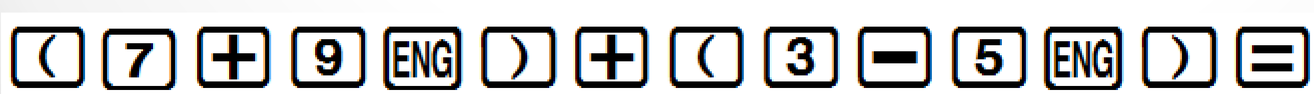
- **Q** $(7,9)+(3,-5)$

$= (7+9i)+(3-5i)$

$= 7+3+(9-5)i$

$=10+4i$





Subtraction of complex numbers

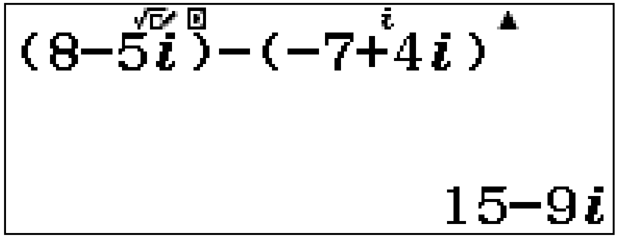
Q $(8,-5)+(-7,4)$

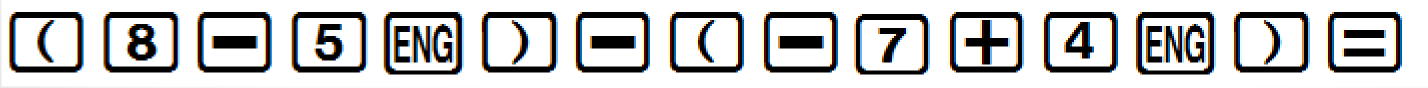
$= (8-5i)-(-7+4i)$

$= 8-5i+7-4i$

$= 8+7-5i-4i$

$= 15-9i$





Multiplication of complex numbers

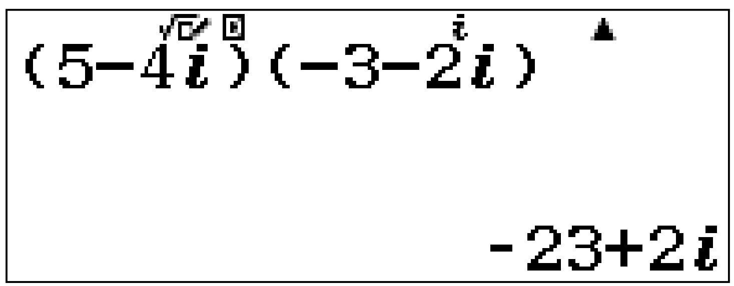
Q $(5,-4)(-3,-2)$

$= (5-4i)(-3-2i)$

$= -15-10i+12i+8i^2$

$= -15+2i-8$

$= -23+2i$



Division of complex numbers

if you found i in denominator then just multiply and divide whole sequence by conjugate of denominator as shown below

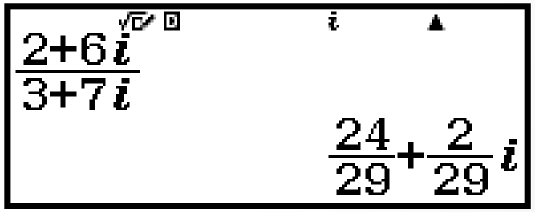
11: $(2, 6) \div (3, 7)$

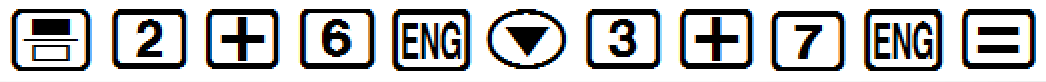
$$\frac{2+6i}{3+7i} = \frac{2+6i}{3+7i} \times \frac{3-7i}{3-7i}$$

$$= \frac{(2+6i)(3-7i)}{(3)^2-(7i)^2}$$

$$= \frac{6+4i-42(-1)}{9+49}$$

$$= \frac{24}{29} + i\frac{2}{29}$$





Multiplicative inverse 'I'

(ii) $(\sqrt{2}, -\sqrt{5})$

$$\textit{Multiplicative Inverse} = \frac{1}{(\sqrt{2}, -\sqrt{5})}$$

$$= \frac{1}{\sqrt{2}-\sqrt{5}i} \times \frac{\sqrt{2}+i\sqrt{5}}{\sqrt{2}+i\sqrt{5}}$$

$$= \frac{\sqrt{2}}{7} + \frac{\sqrt{5}}{7}i$$

$$\frac{1}{\sqrt{2}-\sqrt{5}\,i}$$

$$\frac{\sqrt{2}}{7}+\frac{\sqrt{5}}{7}i$$

[fraction] [1] [▼] [√■] [2] [▶] [−] [√■] [5] [▶] [ENG] [=]

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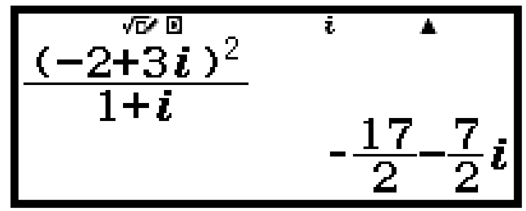
Separate real and imaginary part

16. **Separate into real and imaginary parts (write as a simple complex number):**

(ii) $\dfrac{(-2+3i)^2}{1+i}$



$$\frac{(-2+3i)^2}{1+i}=\frac{4-12i+9i^2}{1+i}\times\frac{1-i}{1-i}$$

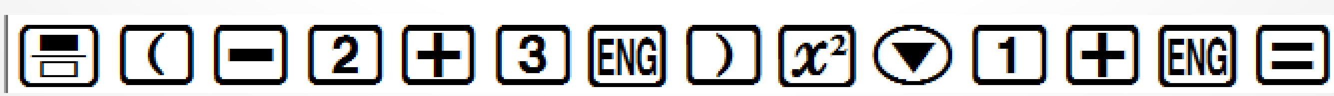
$$=\frac{(4-12i+9(-1))(1-i)}{(1+i)(1-i)}$$

$$=\frac{(4-12i-9)(1-i)}{(1)^2-(i)^2}=\frac{(-5-12i)(1-i)}{1-(-1)}$$

$$=\frac{-5+5i-12i+12i^2}{1+1}$$

$$=\frac{-5-7i+12(-1)}{2}$$

$$=\frac{-17-7i}{2}=\frac{-17}{2}-\frac{7}{2}i$$



Polar form (r,θ)

Example 4: Express the complex number $1+i\sqrt{3}$ in polar form.

(1 + ENG √■ 3 ▶) OPTN ▼ 1 =

$(1+i\sqrt{3})\blacktriangleright r\angle\theta$

$2\angle 60$

Solution: Put $r\cos\theta = 1 \rightarrow (i)$ & $r\sin\theta = \sqrt{3} \rightarrow (ii)$

Squaring & adding (i) & (ii)

$$r^2\cos^2\theta + r^2\sin^2\theta = (1)^2 + (\sqrt{3})^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 1+3$$

$$r^2 = 4$$

$$r = 2$$

Dividing (ii) by (i)

$$\frac{r\sin\theta}{r\cos\theta} = \frac{\sqrt{3}}{1}$$

$$tan\theta = \sqrt{3}$$

$$\theta = Tan^{-1}(\sqrt{3})$$

$$\theta = 60^o$$

$$Thus\ 1+i\sqrt{3} = r(Cos\theta + i\sin\theta)$$

$$= 2(Cos60^o + i\sin 60) \quad Ans$$

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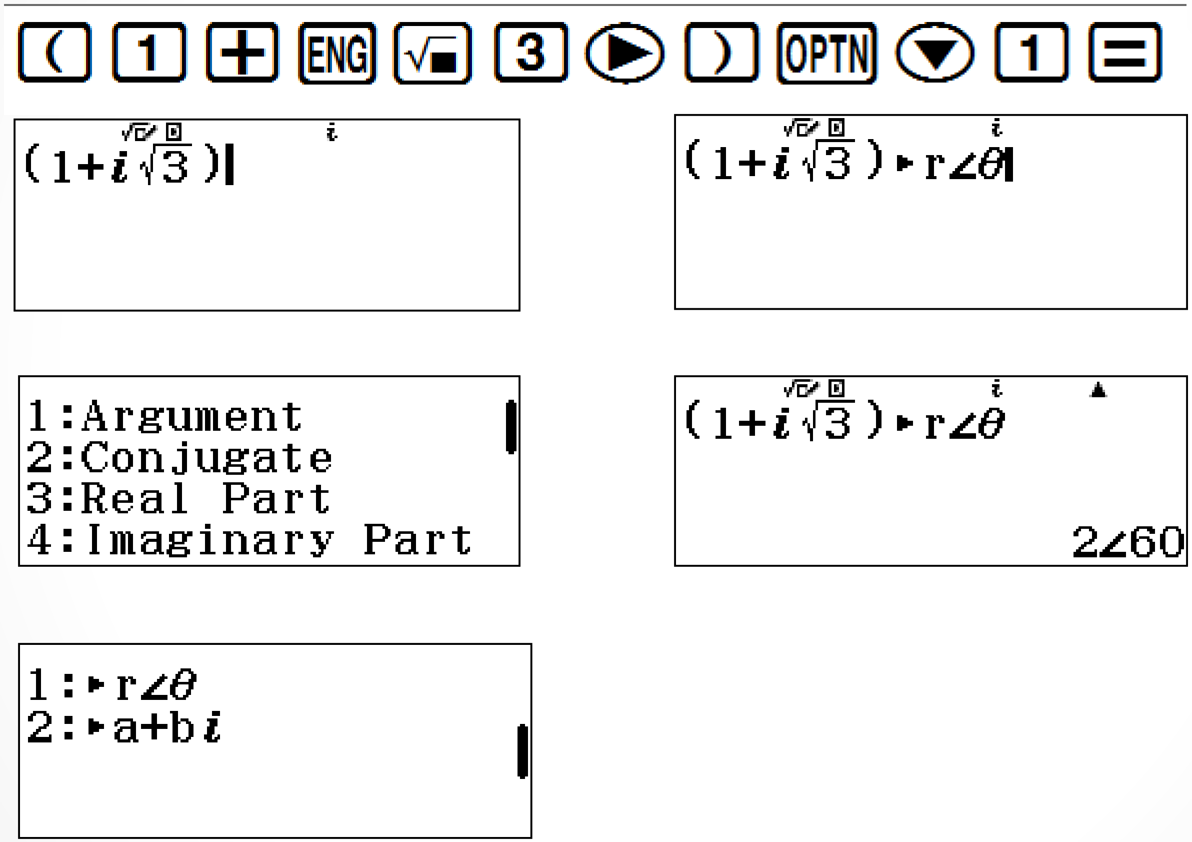
Step by step screen view



Demoiver's theorm

- Statement; $(Cos\theta + i\sin\theta)^n = Cos(n\theta) + i\sin(n\theta)$

Example 5: Find out real and imaginary parts of each of the following complex numbers.

(i) $(\sqrt{3} + i)^3$ **Federal 2009**

Solution

$$Let \quad r\cos\theta = \sqrt{3}, \ \& \ r\sin\theta = 1 \ where$$
$$r^2\cos^2\theta + r^2\sin^2\theta = (\sqrt{3})^2 + (1)^2$$
$$r^2(\cos^2\theta + \sin^2\theta) = 3 + 1$$

$$r = 2$$

$$also \quad \frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$$
$$\tan\theta = \frac{1}{\sqrt{3}}$$
$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$
$$\theta = 30^o$$

$$(\sqrt{3}+1)^3 = [r(\cos\theta + i\sin\theta)]^3 = r^3(\cos\theta + i\sin\theta)^3$$

$$= 2^3(\cos\theta + i\sin\theta)^3 = 8(\cos 3(30^0) + i\sin 3(30^0))$$ **By demoiver's theorem.**

$$(\sqrt{3}+1)^3 = 8\left[\cos 90^o + i\sin 90^o\right]$$

$$= 8[0 + i\,.\,1] = 0 + 8i$$

$$(\sqrt{3}+1)^3 = 8i$$

$(\sqrt{3}+1i) \blacktriangleright r\angle\theta$

2∠30

$(\sqrt{3}+1i)^3 \blacktriangleright r\angle\theta$

8∠90

$(\sqrt{3}+1i)^3$

$8i$

Exercise problem

5. Simplify by expressing ...

i) $5+2\sqrt{-4}$

iii) $\dfrac{2}{\sqrt{5}+\sqrt{-8}}$

iv) $\dfrac{3}{\sqrt{6}-\sqrt{-12}}$

6. Show that $\forall\, z \in C$

i) $z^2+\bar{z}^2$ is a real number.

ii) $(z-\bar{z})^2$ is a real nu

7. Simplify the following

i) $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3$

ii) $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^3$

iii) $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

iv) $(a+bi)^2$

v) $(a+bi)^{-2}$

vi) $(a+bi)^3$

vii) $(a-bi)^3$

viii) $\left(3-\sqrt{-4}\right)^{-3}$

Common mistake

$$\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)^3$$

1

$$\tan^{-1}(-\sqrt{3})$$

-60

$$\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)\blacktriangleright r\angle\theta$$

$1\angle120$

=[1(cos120 +isin120)] ^{2}
=cos360+ison360
=1+0
=1

$$\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^3$$

1

$$\tan^{-1}(\sqrt{3})$$

60

$$\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)\blacktriangleright r\angle\theta$$

$1\angle-120$

=[1(cos-120 +isin-120)] ^{2}
=cos360-ison360
=1-0
=1

How to graph numbers on complex plane

(i) $\mathbf{2+3i}$

Sol. $2+3i$ *Compare with* $x+iy$

Here $x=2,\ y=-3$

(ii) $\mathbf{2-3i}$

Sol. $2-3i$ *Compare with* $x+iy$

$x=2,\quad y=-3$

(iii) $\mathbf{-2-3i}$

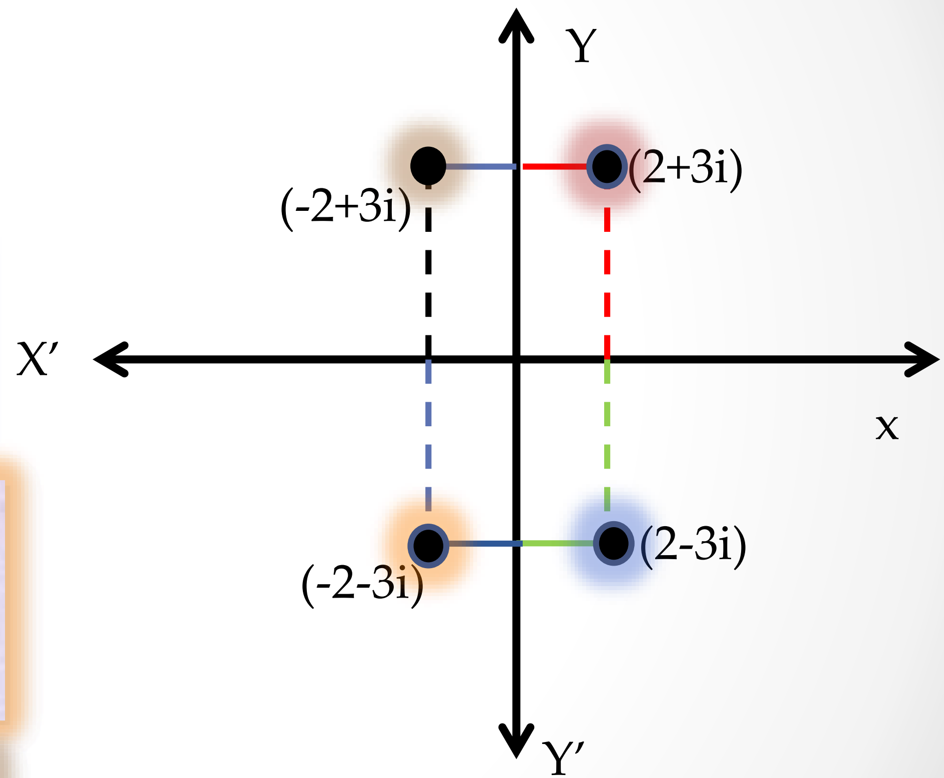
Sol. *Compare with* $x+iy$

$x=-2,\quad y=-3$

(iv) $\mathbf{-2+3i}$

Sol. $-2+3i$ *Compare with* $x+iy$

$x=-2,\quad y=3$



5. Simplify by expressing in the form $a+bi$

(iv) $\dfrac{3}{\sqrt{6}-\sqrt{-12}}$

$$\frac{3}{\sqrt{6}-\sqrt{-12}}=\frac{3}{\sqrt{6}-i\sqrt{12}}\times\frac{\sqrt{6}+i\sqrt{12}}{\sqrt{6}+i\sqrt{12}}$$

$$=\frac{3\sqrt{6}+i3\sqrt{12}}{(\sqrt{6})^2-(i\sqrt{12})^2}=\frac{3\sqrt{6}+i3\sqrt{12}}{6-(-12)}$$

$$=\frac{3\sqrt{6}+i3\sqrt{12}}{6+12}=\frac{3(\sqrt{6}+i\sqrt{12})}{18}$$

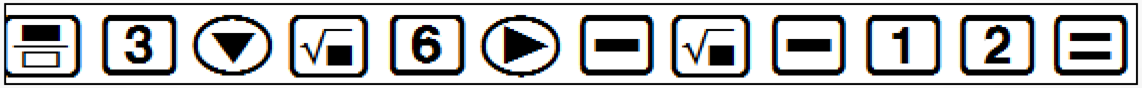
$$=\frac{\sqrt{6}}{6}+i\frac{\sqrt{4\times3}}{6}=\frac{\sqrt{6}}{\sqrt{6}\sqrt{6}}+i\frac{2\sqrt{3}}{6}$$

$$=\frac{1}{\sqrt{6}}+\frac{i\sqrt{3}}{3}=\frac{1}{\sqrt{6}}+\frac{i\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$=\frac{1}{\sqrt{6}}+\frac{i}{\sqrt{3}}$$

$$\frac{3}{\sqrt{6}-\sqrt{-12}}$$

$$\frac{\sqrt{6}}{6}+\frac{\sqrt{3}}{3}i$$



ii) $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^3$

$= \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^2\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= \left[\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}i\right)^2-2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)\right]\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= \left(\frac{1}{4}-\frac{3}{4}+\frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= \left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= \left(-\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}i\right)^2$

$= \frac{1}{4}-\frac{3}{4}i^2=\frac{1}{4}+\frac{3}{4}=\frac{1+3}{4}=\frac{4}{4}=1$

Ans

$\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^3$

1

$\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^2$

$\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$

$-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$

1