Solving Samples of Math Problems

Using

CASIO FX-CG50 CALCULATOR

AP-EXAM

Done By Casio Middle East - GAKUHAN

AP Exams are standardized exams designed to measure how well you've mastered the content and skills of a specific AP course. Most AP courses have an end-of-year paper-and-pencil exam, but a few courses have different ways to assess what you've learned.

Advanced Placement Calculus (also known as AP Calculus) is a set of two distinct Advanced Placement calculus courses and exams offered by College Board. AP Calculus AB covers limits, derivatives, and integrals. AP Calculus BC covers all AP Calculus AB topics plus additional topics (including more integration techniques such as integration by parts, Taylor series, parametric equations, polar coordinate functions, and curve interpolations).

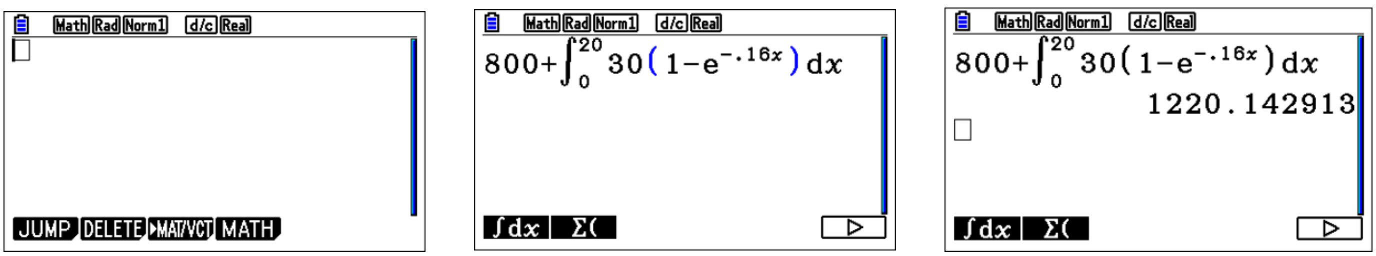
Multiple-Choice, Section I Part A	Multiple-Choice, Section I Part B	Free-Response, Section II Part A	Free-Response, Section II Part B	
# of Questions	30	15	2	4
Time Allowed	60 minutes	45 minutes	30 minutes	60 minutes
Calculator Use	No	Yes	Yes	No

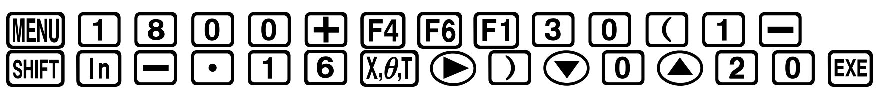
Q.1. Water is pumped into a tank at a rate of $r(t) = 30\left(1 - e^{-0.16t}\right)$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A) 380 gallons
(B) 420 gallons
(C) 829 gallons
(D) 1220 gallons
(E) 1376 gallons

Answer : (D) 1220 gallons

Because the rate of change , at t=0 the tank was 800 gallon , at t=20 how much water ?





Q.2. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{\left(x^4 - 2x^2 + 1\right)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

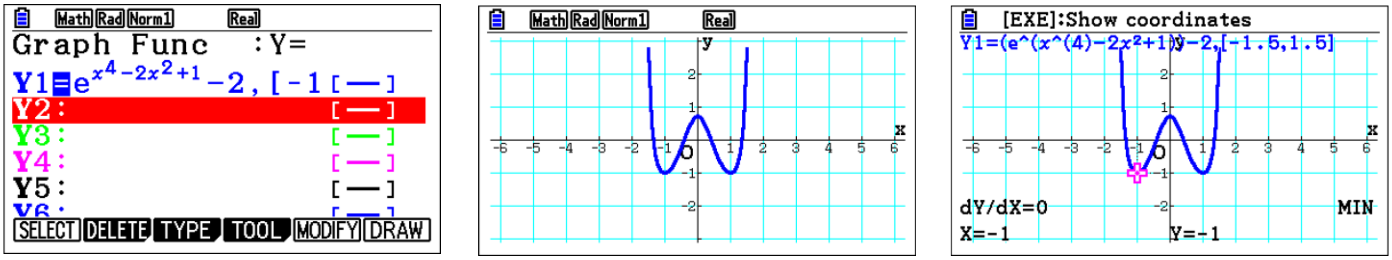
(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

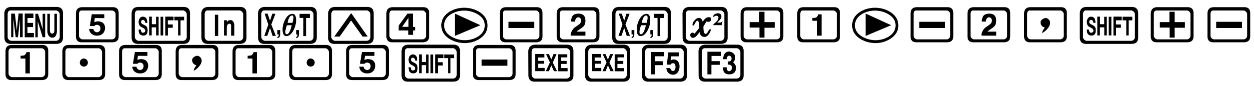
Answer : (D) (-1.5,-1) and ((0,1)

Note: f(x) is concave down when f''(x) is negative.

$$f^{\wedge\prime\prime}(x) = e^{\wedge}((x^{\wedge}4 - 2x^{\wedge}2 + 1))\,(4x^{\wedge}3 - 4x)$$

Or f'(x) slope is negative (decreasing)

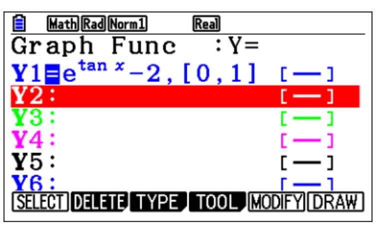




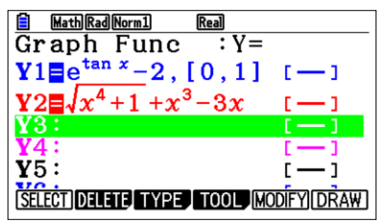
Q.3 If $f'(x) = \sqrt{x^4+1} + x^3 - 3x$, then f has a local maximum at $x =$

(A) -2.314 (B) -1.332 (C) 0.350 (D) 0.829 (E) 1.234

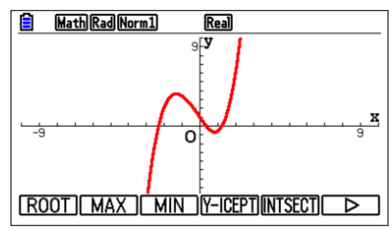
Note: f(x) has local maximum when f'(x) changes from positive to negative.



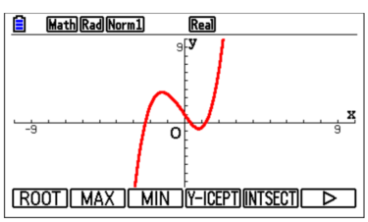
1- select Graph mode

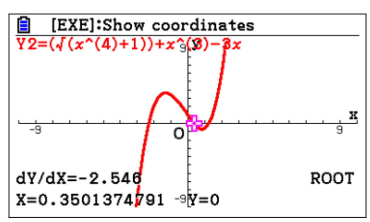


2- write the function



3- draw the function





4- G-Solve to see the roots

Answer : (D) (-1.5,-1) and ((0,1)

Q.4. Which of the following is an equation for a line tangent to the graph of $f(x) = e^{2x}$ when $f'(x) = 10$?

(A) $y = 10x - 8.05$

(B) $y = x - 8.05$

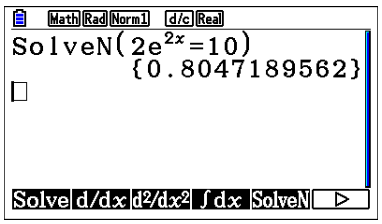
(C) $y = x - 3.05$

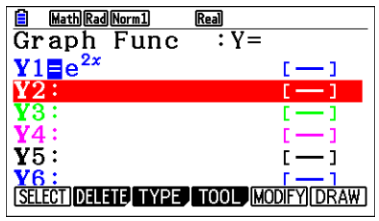
(D) $y = 10x - 11.5$

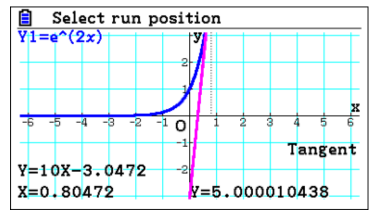
(E) $y = 10x - 3.05$

Answer : (E) y=10x-3.05

Note: we need the point to write the equation by using f'(x) = 10
find x then substitute to find y







OPTN F4 F5 2 SHIFT ln 2 X,θ,T ▶ SHIFT • 1 0) EXE EXIT
MENU 5 SHIFT ln 2 X,θ,T EXE EXE F4 F2 • 8 0 4 7 2 EXE EXE

Q.5. What is the area of the region bounded by $y = \sin x$, $y = \frac{1}{4}x - 1$, and the y-axis?

(A) 0.772

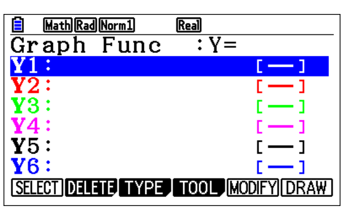
(B) 2.815

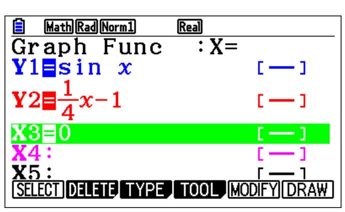
(C) 3.926

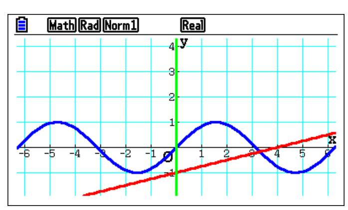
(D) 5.552

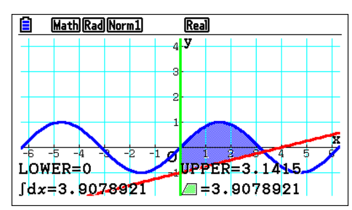
(E) 34.882

Answer : (c) 3.926









sin X,θ,T EXE ▤ 1 ▼ 4 ▶ X,θ,T
EXE F3 F4 0 EXE F6 F5 F6 F3 F4 ▶ ▶ EXE

Q.6. The function f whose derivative is given by $f'(x) = 5x^3 - 15x + 7$ has a local maximum at $x =$

(A) -1.930

(B) -1.000

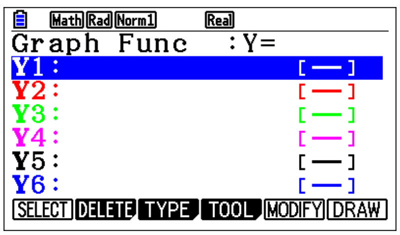
(C) 0.511

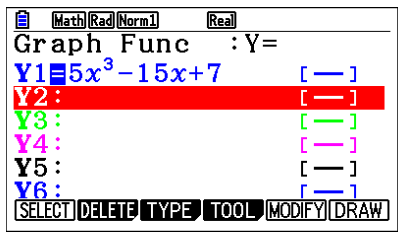
(D) 1.000

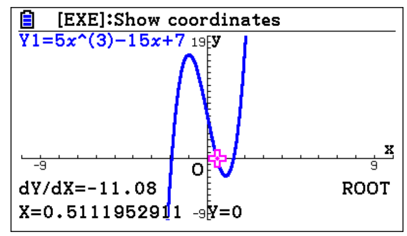
(E) 1.419

Answer : (B) 0.511

Note: local maximum is a point which is the function goes increasing then decreasing for f(x), and the graph is positive then negative for f'(x)







Q.7. Let f be the function given by $f(x) = \dfrac{3x^3}{e^x}$. For what value of x is the slope of the line tangent to f equal to -1.024?

(A) -9.004

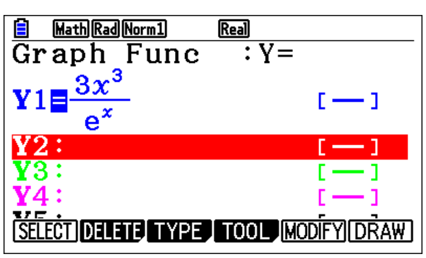
(B) -4.732

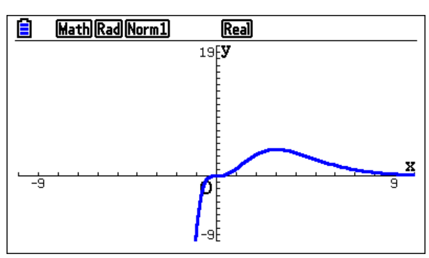
(C) 1.029

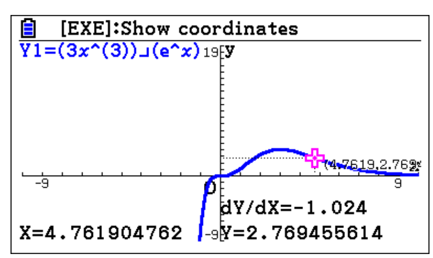
(D) 1.277

(E) 4.797

Answer : (E) 4.797







Q.8. The graph of the function $y = x^5 - x^2 + \sin x$ changes concavity at $x =$

(A) 0.324

(B) 0.499

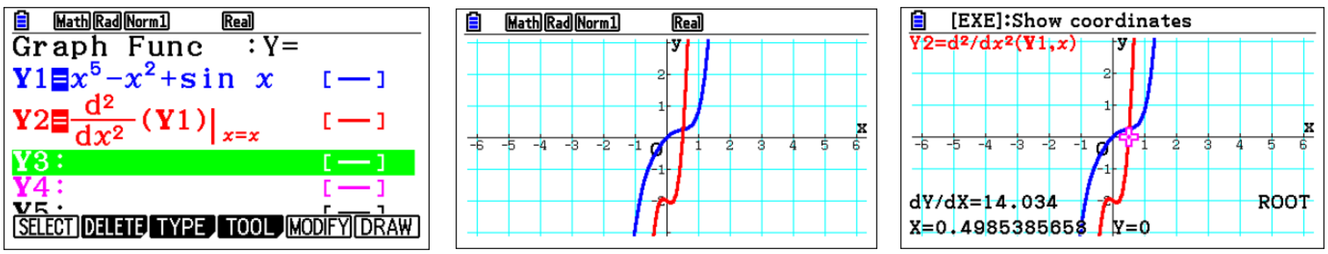
(C) 0.506

(D) 0.611

(E) 0.704

Answer : (B) 0.499

Note: second derivative graph shows the concavity clearly at the root.



MENU 5 X,θ,T ^ 5 ▶ x² + sin X,θ,T EXE OPTN
F2 F2 F1 1 ▶ X,θ,T EXE EXE F5 F1 ▼ EXE

Q.9. If $f(x) = \tan^2 x + \sin x$, then $f'\left(\frac{\pi}{4}\right) =$

(A) $\frac{4+\sqrt{2}}{2}$

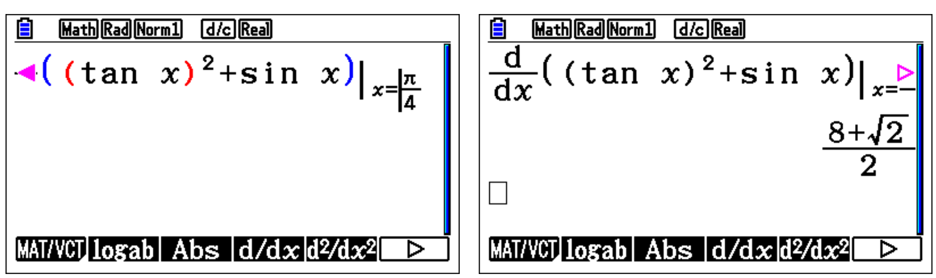
(B) $\frac{2+\sqrt{2}}{2}$

(C) $\frac{8+\sqrt{2}}{2}$

(D) $\frac{8-\sqrt{2}}{2}$

(E) $\frac{4-\sqrt{2}}{2}$

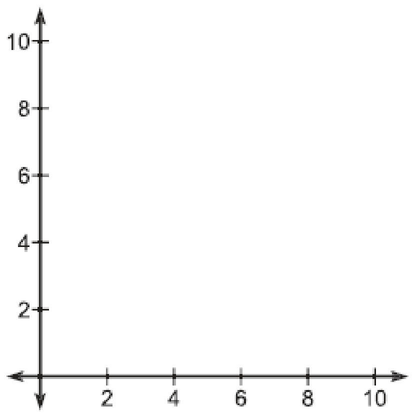
Answer : (C)



F4 F4 (tan X,θ,T) x² + sin X,θ,T ▶ SHIFT ×10^x ÷ 4

Q.10. Let R be the region bound by $y = 2x^2 - 8x + 11$ and $y = x^2 - 4x + 10$.

(a) Sketch the region on the axes provided.



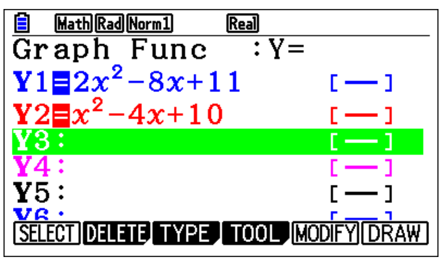
(b) Determine the area of R.

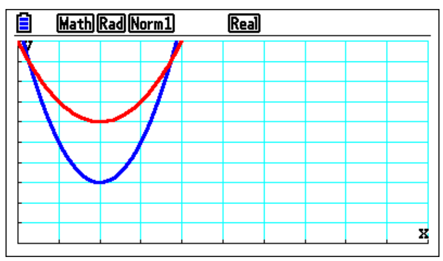
1. Drawing the functions

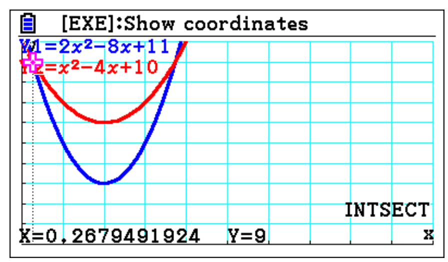
2. determine the intersection points

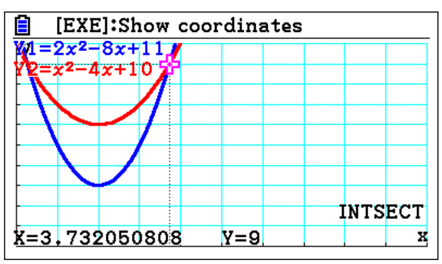
3. Calculate the area

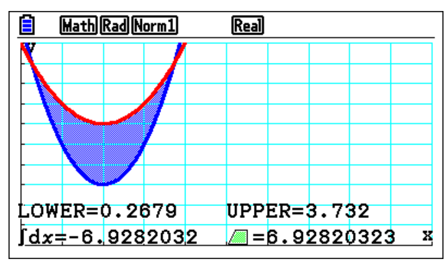
Note: after sketching the graph we need to determine the intersection points to find the area and volume .

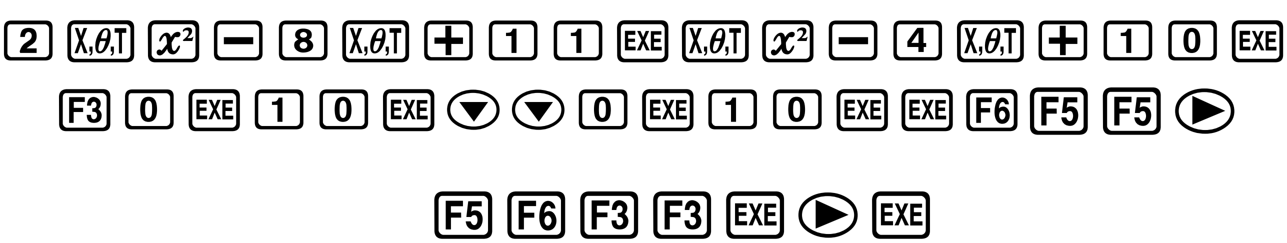










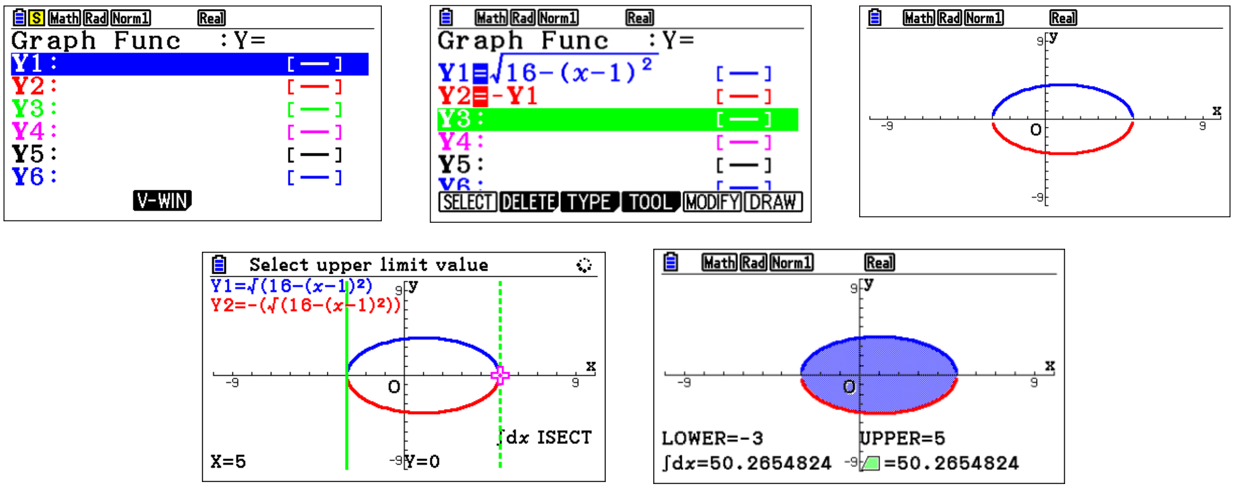


Answer : (6.928)

Q.11. Let f be the functions given by:

$$f(x) = \sqrt{16 - (x-1)^2}$$

Find the area of the region enclosed by the graphs of f(x) and -f(x).



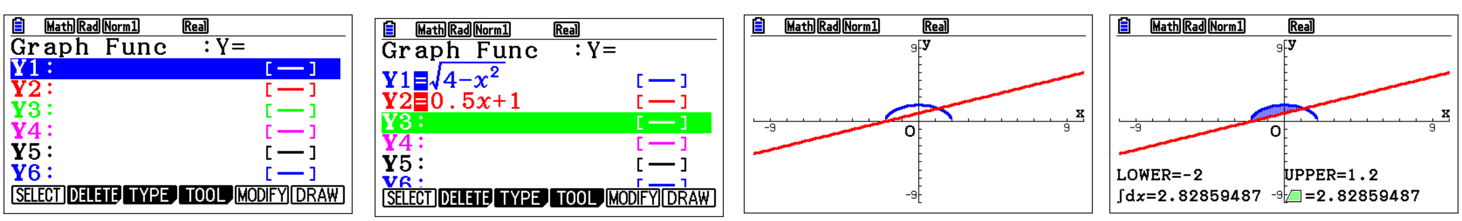
MENU 5 SHIFT x^2 1 6 − (X,θ,T − 1) x^2 EXE −
F1 1 EXE EXE F3 F3 EXIT F6 F5 F6 F3 F3 EXE ▶ EXE

Answer : (50.265)

Q.12. t f and g be the functions given by:

$$f(x) = \sqrt{4 - x^2}$$
$$g(x) = 0.5x + 1$$

Let R be the region in the first and second quadrants enclosed by the graphs of f and g. Find the area of R.

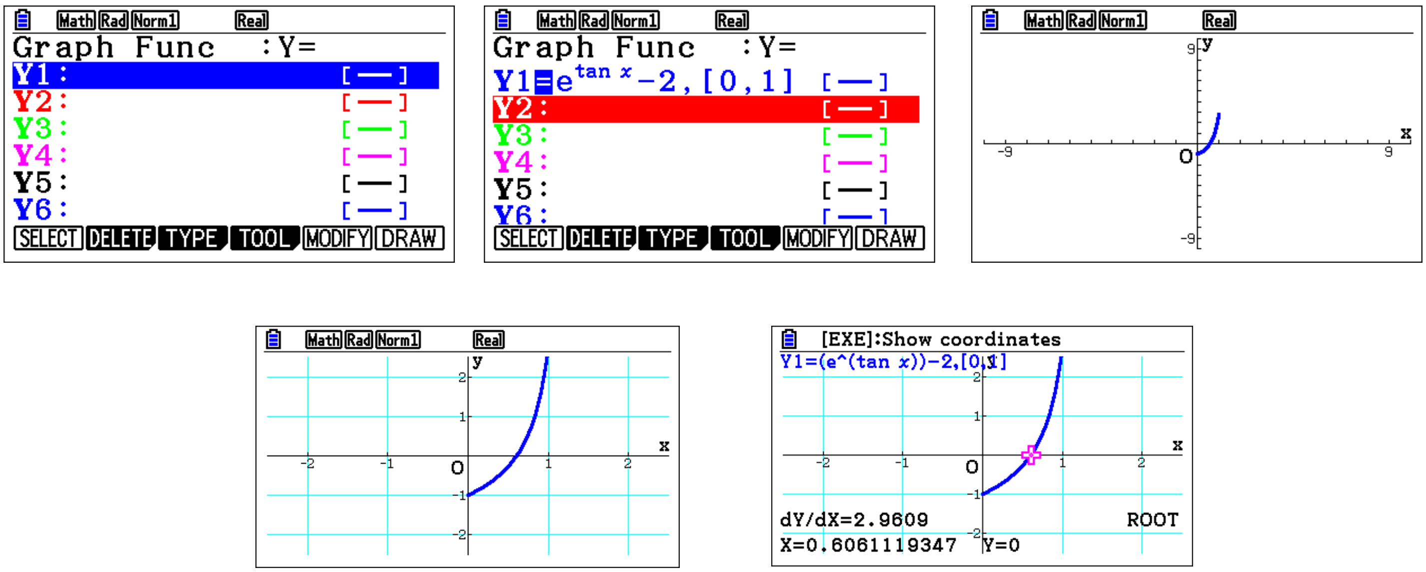


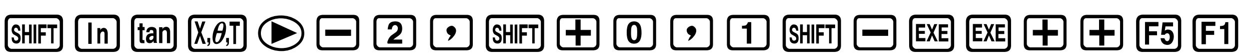
SHIFT x^2 4 − X,θ,T x^2 EXE 0 • 5 X,θ,T + 1 EXE EXE F5 F6 F3 F3 EXE ▶ EXE

Answer : (2.828)

Q.13. The graph of $y = e^{\tan x} - 2$ crosses the x-axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

(A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747



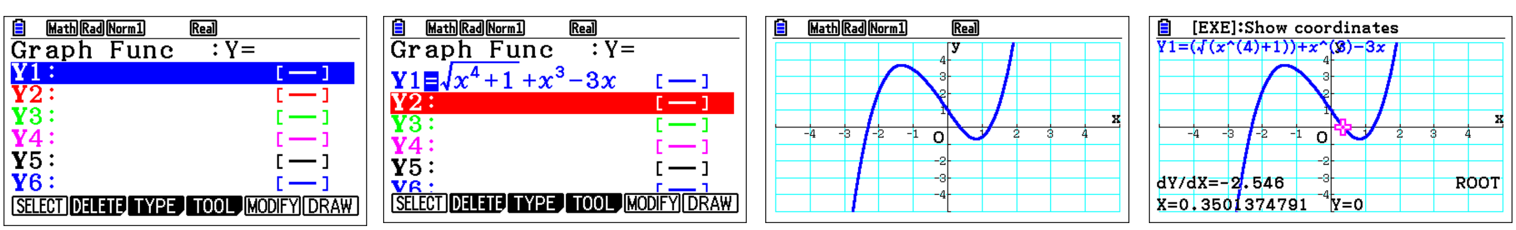


Answer : (D) 2.961

Q.14. If $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x$, then f has a local maximum at $x =$

(A) -2.314 (B) -1.332 (C) 0.350 (D) 0.829 (E) 1.234

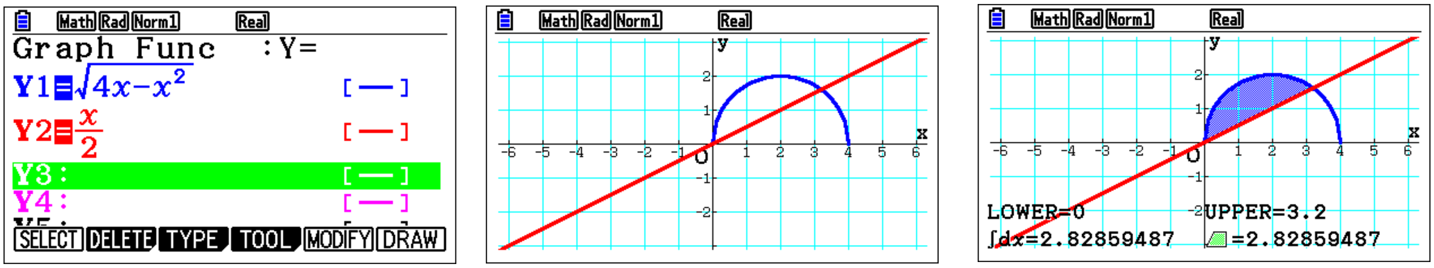
Note: the maximum local are exist when the curve increasing then decreasing for f(x) , and if the curve above x-axis then goes down x-axis for f’(x)



Answer : (C) 0.350

Q.15. What is the area of the region enclosed by the graphs of $y = \sqrt{4x - x^2}$ and $y = \dfrac{x}{2}$?

(A) 1.707 (B) 2.829 (C) 5.389 (D) 8.886 (E) 21.447

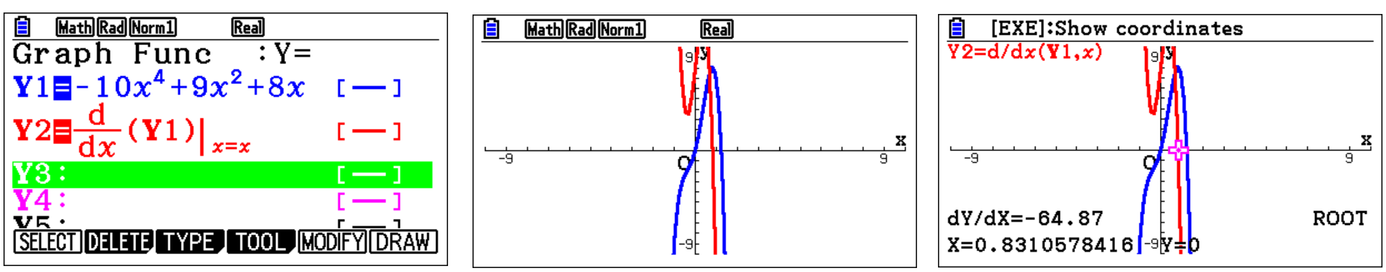


[MENU] [5] [SHIFT] [x^2] [4] [X,θ,T] [x^2] [EXE] [X,θ,T] [▭/▭] [2] [EXE]
[EXE] [F3] [F1] [EXIT] [F6] [F5] [F6] [F3] [F3] [EXE] [▶] [EXE]

Answer : (B) 2.829

Q.16. A particle moves along the x-axis so that its position at time $t > 0$ is given by $x(t)$ and $\dfrac{dx}{dt} = -10t^4 + 9t^2 + 8t$. The acceleration of the particle is zero when $t =$

(A) 0.387 (B) 0.831 (C) 1.243 (D) 1.647 (E) 8.094



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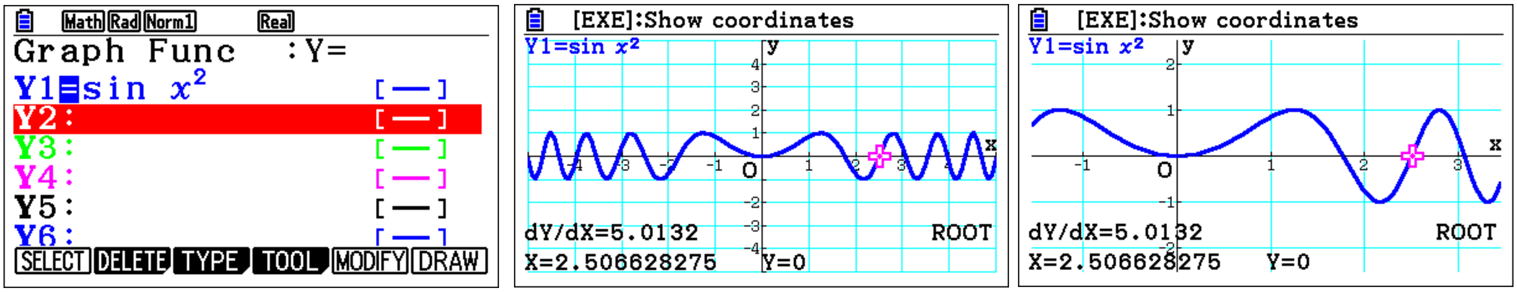
[EXE] [OPTN] [F2] [F1] [F1] [1] [▶] [X,θ,T] [F5] [F1] [▼] [EXE] [EXE] [EXE]

Answer : (B) 0.831

Q.17. The first derivative of the function f is given by $f'(x) = \sin(x^2)$. At which of the following values of x does f have a local minimum?

(A) 2.507 (B) 2.171 (C) 1.772 (D) 1.253 (E) 0



Answer : (A) 2.507

Q.18. Let f be the function given by $f(x) = \dfrac{3x^3}{e^x}$. For what value of x is the slope of the line tangent to f equal to -1.024?

(A) -9.004

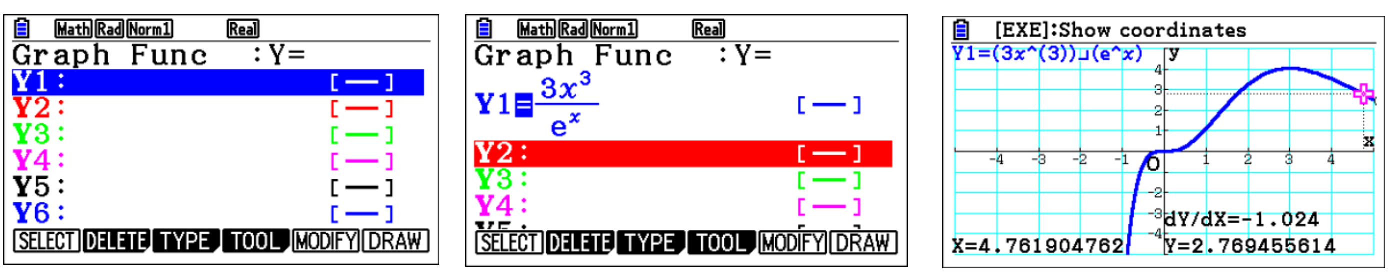
(B) -4.732

(C) 1.029

(D) 1.277

(E) 4.797

Answer : (E) 4.797



Q.19. $\int_1^5 \left(\frac{3x}{x^3}\right) dx =$

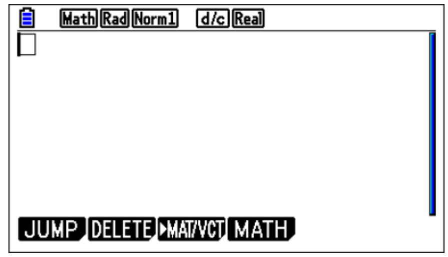
(A) $-\frac{18}{5}$

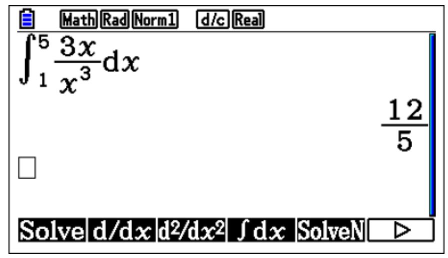
(B) $-\frac{72}{25}$

(C) $\frac{124}{125}$

(D) $\frac{126}{125}$

(E) $\frac{12}{5}$





Answer : (E) $\frac{12}{5}$

MENU 1 OPTN F4 F4 3 X,θ,T ▭ X,θ,T ∧ 3 ▶ ▶ ▶ 1 ▲ 5 EXE

Q.20. The graph of $y = 3x^3 - 2x^2 + 6x - 2$ is decreasing for which interval(s)?

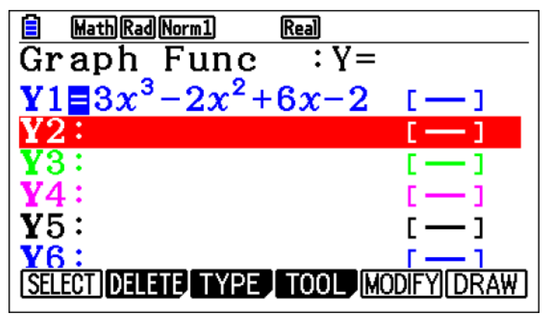
(A) $\left(-\infty, \frac{2}{9}\right)$

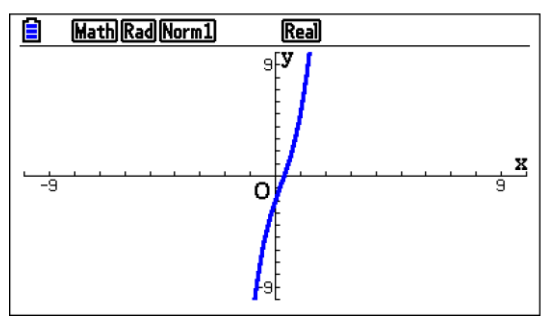
(B) $\left(\frac{2}{9}, \infty\right)$

(C) $\left[0, \frac{2}{9}\right]$

(D) $(-\infty, \infty)$

(E) None of the above



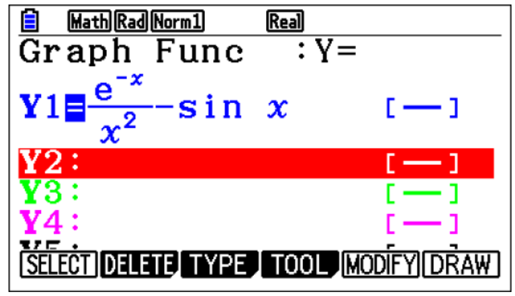


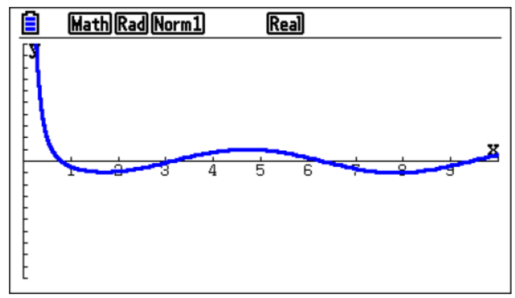
MENU 5 3 X,θ,T ∧ 3 ▶ − 2 X,θ,T x² + 6 X,θ,T − 2 EXE EXE

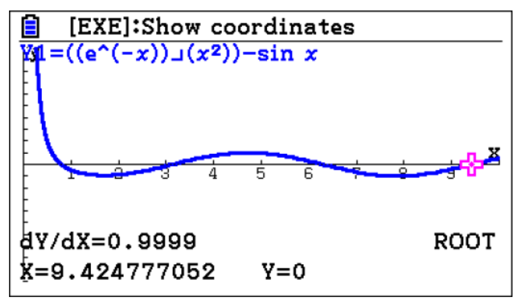
Answer : (E) Non of the above

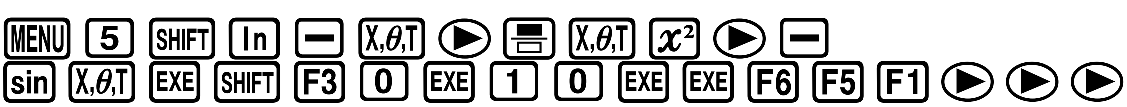
Q.21. The first derivative of a function, f, is given by $f'(x) = \frac{e^{-x}}{x^2} - \sin x$. How many critical values does f have on the open interval $(0,10)$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five







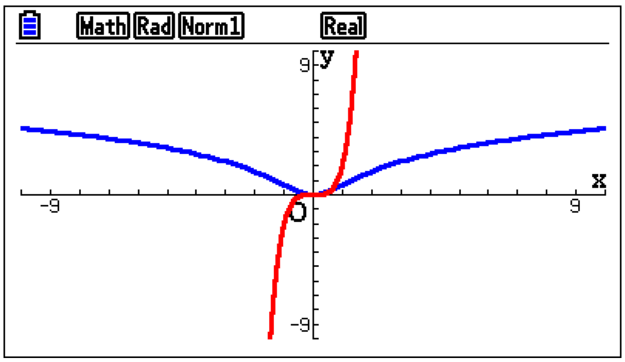


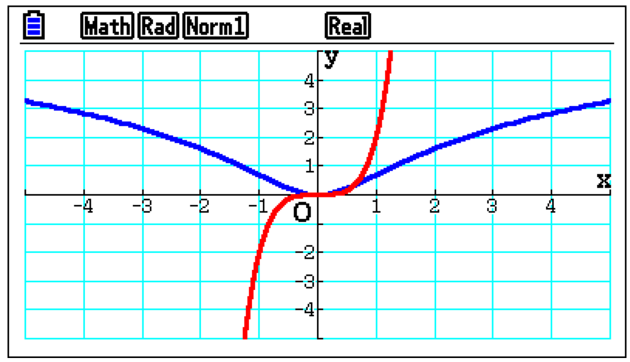
Answer : (D) Four (intersection with x-axis)

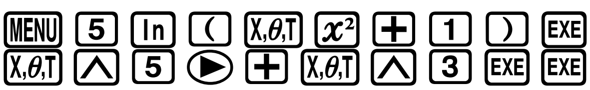
Q.22. Let f be the function defined by $f(x) = \ln(x^2 + 1)$, and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at $x = 2$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

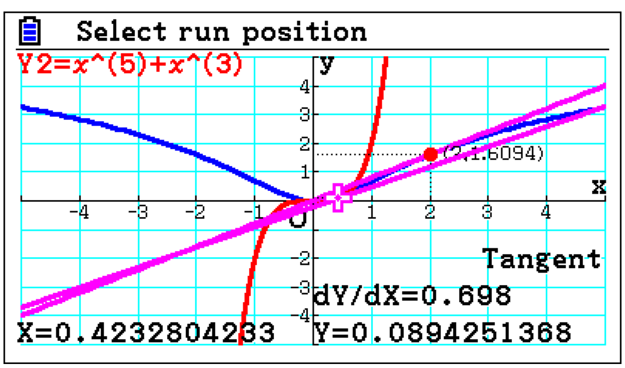
(A) 0.246 (B) 0.430 (C) 0.447 (D) 0.790

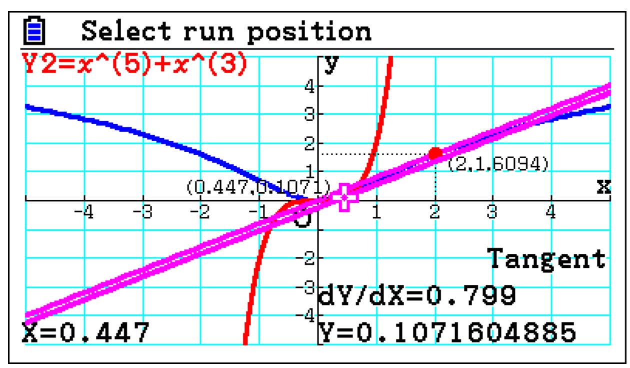
Graph the two functions then for f(x) sketch the tangent line after that you can sketch parallel tangent line for g(x).

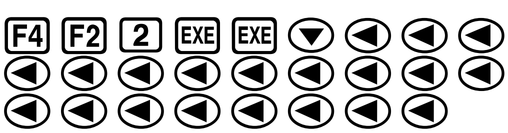












Answer : (C) 0.447

Q.23. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\dfrac{t}{100}\right)^3\left(1 - \dfrac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

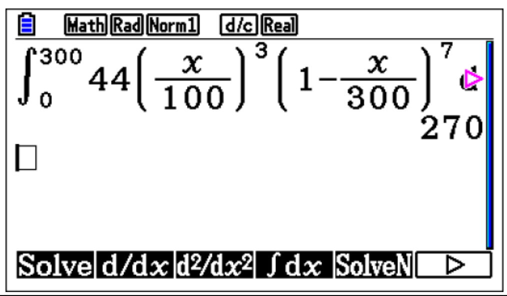
(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

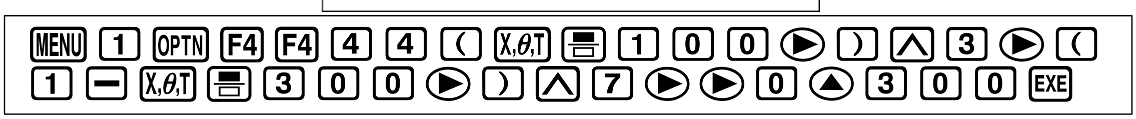
(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

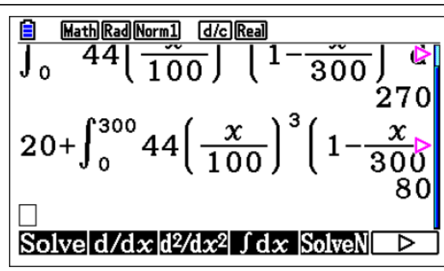
(a) During the interval the number of people are the integration of rate.



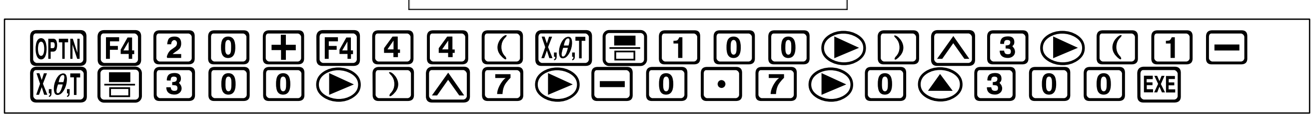
Answer : 270 people



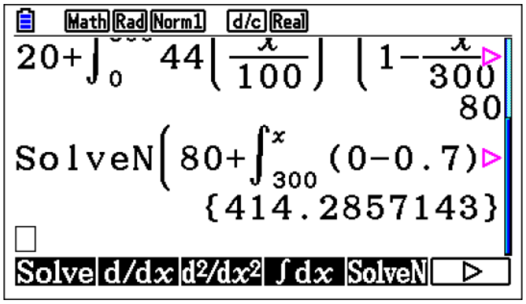
(b) The function r(t) is rate in and 0.7 us rate out and at t=0 there are 20 people .



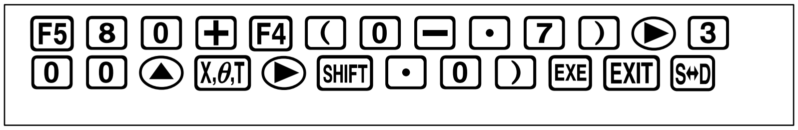
Answer : 80 people



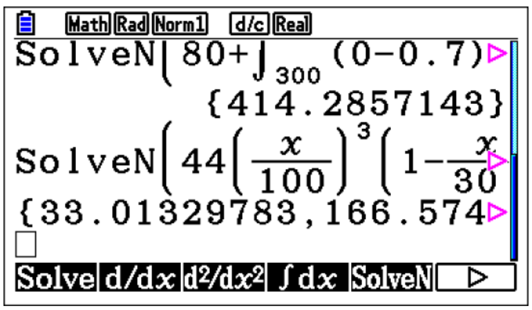
(c) At t=300 there are 80 people and for t >300 the r(t) =0 but the rate out r(t) = 0.7 ,we need first t with no people

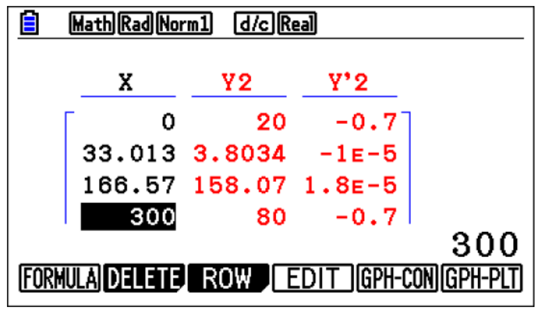


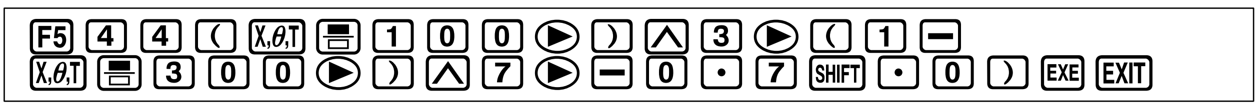
Answer : t= 414.3 sec

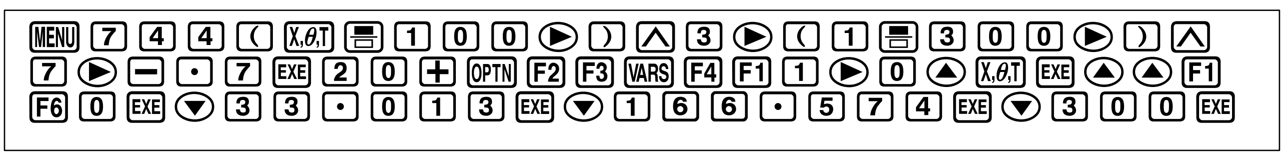


(d) Number of people is minimum so we need to find the critical points (r(t) – 0.7 =0) and make a table to compare the critical points.









Answer : Minimum people at t= 33 & maximum at t= 166.5

Q.24. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

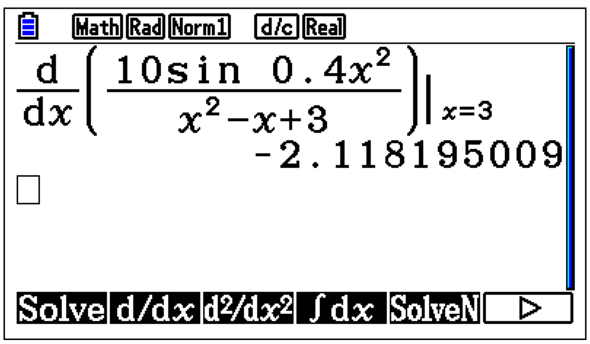
(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

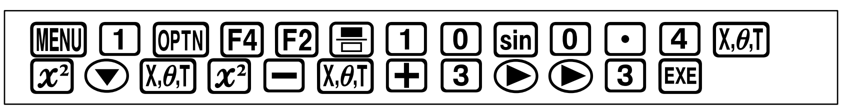
(c) Evaluate $\int_0^{3.5} v(t)\, dt$, and evaluate $\int_0^{3.5} |v(t)|\, dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

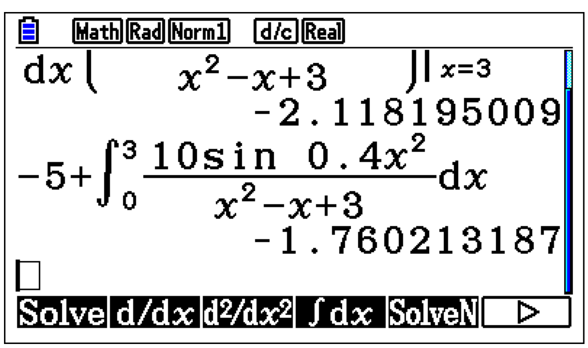
(a) Acceleration a(t) =v'(t)



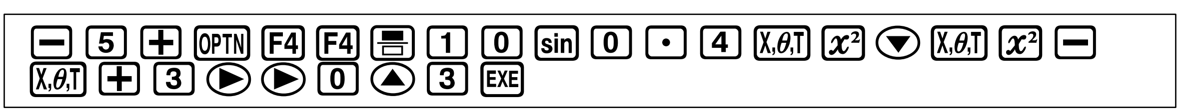
Answer a = -2.118



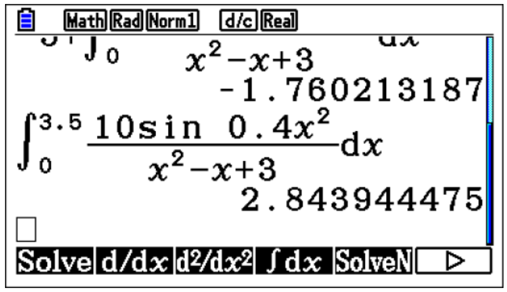
(b) Position x= -5 at t=0 , x(t) = ∫ v(t) dt



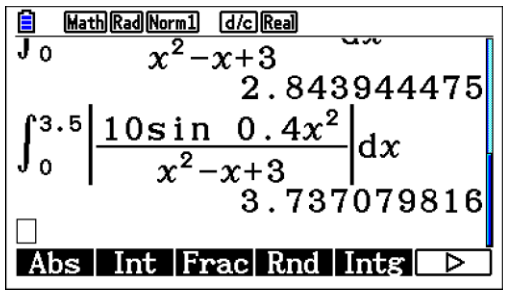
Answer x = -1.76



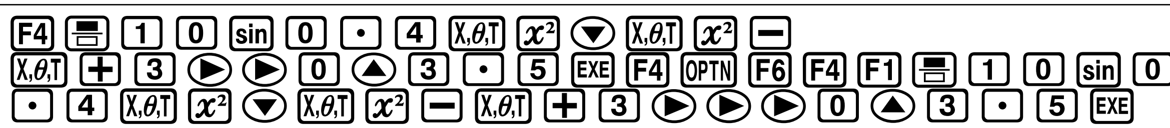
(c) Evaluations , the particle displacement from 0 to 3.5 is 2.844 , and the particle distance traveled is 3.737



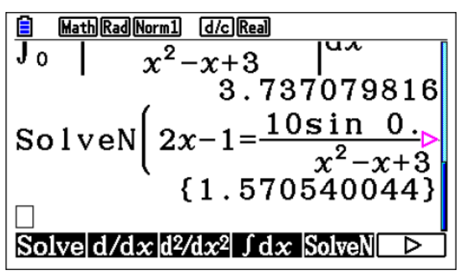
Answer: 2.844



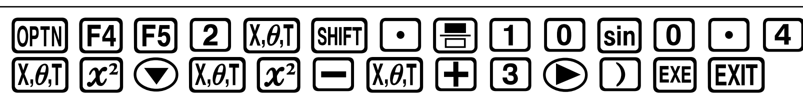
Answer : 3.737



(d) V1(t) = V2(t) , V2(t)=2t-1

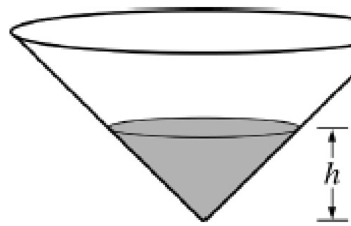


Answer : t =1.57



Q.25. Free Response: Section II, Part A

A graphing calculator is required for problems on this part of the exam.



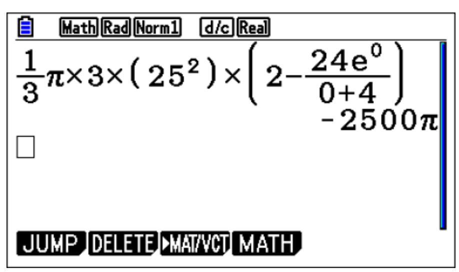
The height of the water in a conical storage tank, shown above, is modeled by a differentiable function h, where $h(t)$ is measured in meters and t is measured in hours. At time $t = 0$, the height of the water in the tank is 25 meters. The height is changing at the rate $h'(t) = 2 - \frac{24e^{-0.025t}}{t+4}$ meters per hour for $0 \le t \le 24$.

(a) When the height of the water in the tank is h meters, the volume of water is $V = \frac{1}{3}\pi h^3$. At what rate is the volume of water changing at time $t = 0$? Indicate units of measure.

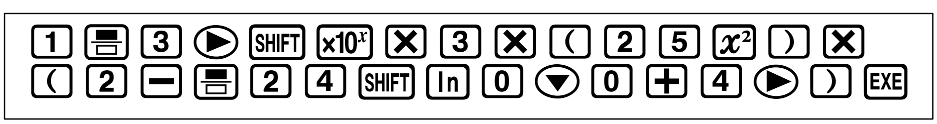
(b) What is the minimum height of the water during the time period $0 \le t \le 24$? Justify your answer.

(a) At t=0 , h=25

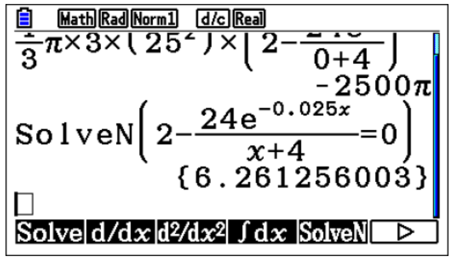
$$\frac{dv}{dt} = \frac{1}{3} \times \pi \times 3h^2 \frac{dh}{dt}$$

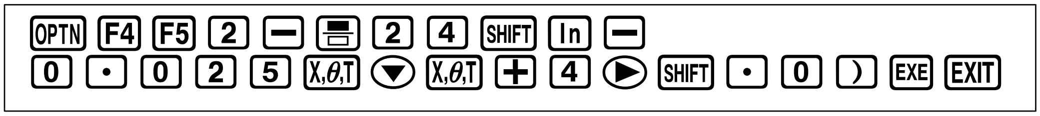


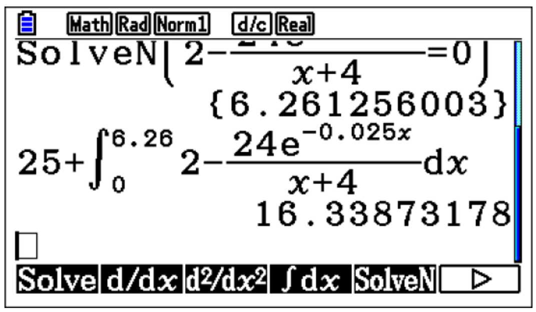
Answer : $\frac{dv}{dt} = -2500\pi$



(b) The absolute minimum must be a critical point for the endpoint.
So we need to solve h’(t)=0 to get t = 6.26 ,
and at t=0 h=25 . the minimum is 25 + the integration from 0 to 6.25







Answer : h =16.34