

Solving Samples of Math Problems
Using
CASIO FX-CG50 CALCULATOR

AP-EXAM

Done By Casio Middle East - GAKUHAN

**Support
Classroom with
Technology**

CASIO calculators - smart educational tools
for ideal educational environments.



AP Exams are standardized exams designed to measure how well you've mastered the content and skills of a specific AP course. Most AP courses have an end-of-year paper-and-pencil exam, but a few courses have different ways to assess what you've learned.

Advanced Placement Calculus (also known as AP Calculus) is a set of two distinct Advanced Placement calculus courses and exams offered by College Board. AP Calculus AB covers limits, derivatives, and integrals. AP Calculus BC covers all AP Calculus AB topics plus additional topics (including more integration techniques such as integration by parts, Taylor series, parametric equations, polar coordinate functions, and curve interpolations).

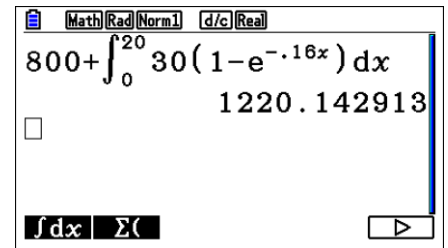
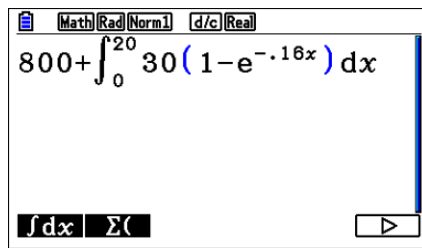
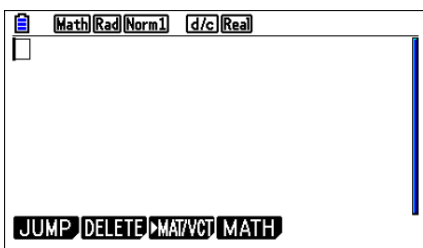
	Multiple-Choice, Section I Part A	Multiple-Choice, Section I Part B	Free-Response, Section II Part A	Free-Response, Section II Part B	
# of Questions		30	15	2	4
Time Allowed		60 minutes	45 minutes	30 minutes	60 minutes
Calculator Use		No	Yes	Yes	No

Q.1. Water is pumped into a tank at a rate of $r(t) = 30(1 - e^{-0.16t})$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

- (A) 380 gallons
- (B) 420 gallons
- (C) 829 gallons
- (D) 1220 gallons
- (E) 1376 gallons

Answer : (D) 1220 gallons

Because the rate of change , at t=0 the tank was 800 gallon , at t=20 how much water ?



Q.2. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{(x^4 - 2x^2 + 1)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

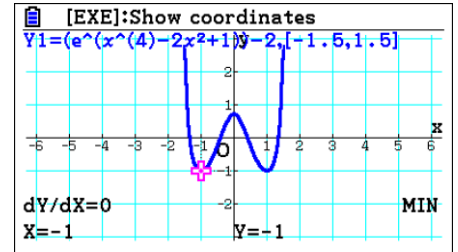
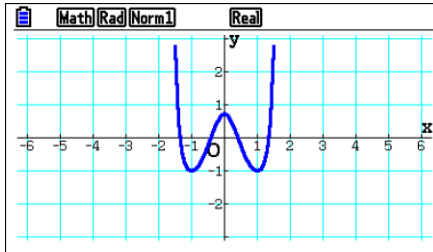
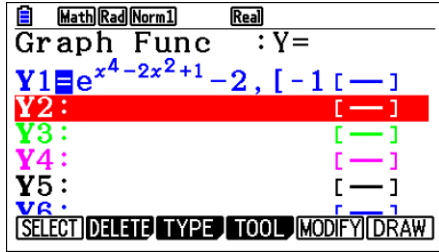
- (A) $(-0.418, 0.418)$ only
- (B) $(-1, 1)$
- (C) $(-1.354, -0.409)$ and $(0.409, 1.354)$
- (D) $(-1.5, -1)$ and $(0, 1)$
- (E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

Answer : (D) $(-1.5, -1)$ and $(0, 1)$

Note: $f(x)$ is concave down when $f''(x)$ is negative.

$$f''(x) = e^{(x^4 - 2x^2 + 1)} (4x^3 - 4x)$$

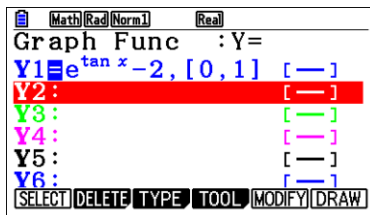
Or $f'(x)$ slope is negative (decreasing)



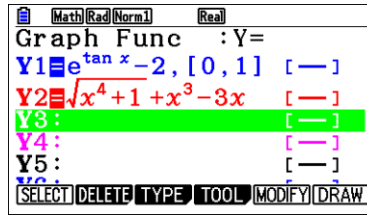
MENU 5 SHIFT In X,θ,T ^ 4 ► - 2 X,θ,T x² + 1 ► - 2 , SHIFT + -
 1 • 5 , 1 • 5 SHIFT - EXE EXE F5 F3

- Q.3** If $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x$, then f has a local maximum at $x =$
 (A) -2.314 (B) -1.332 (C) 0.350 (D) 0.829 (E) 1.234

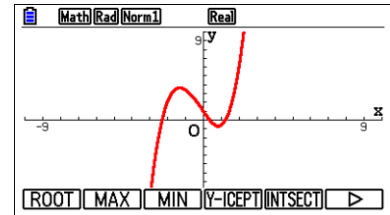
Note: $f(x)$ has local maximum when $f'(x)$ changes from positive to negative.



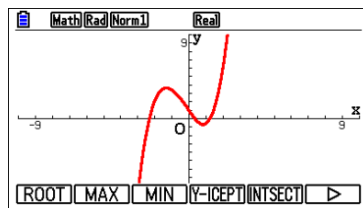
1- select Graph mode



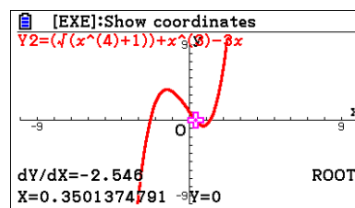
2- write the function



3- draw the function



4- G-Solve to see the roots



MENU 5 ▼ SHIFT x² X,θ,T ^ 4 ► + 1 ► + X,θ,T ^ 3 ► - 3 X,θ,T EXE

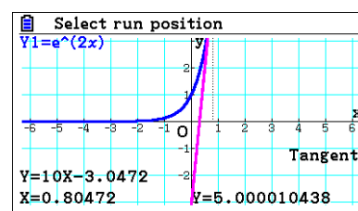
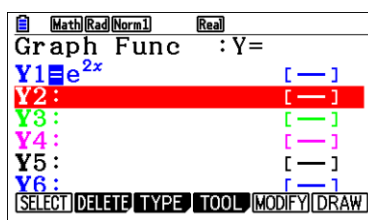
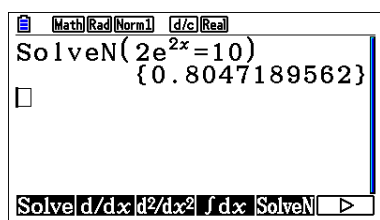
Answer : (D) (-1.5, -1) and (0,1)

Q.4. Which of the following is an equation for a line tangent to the graph of $f(x) = e^{2x}$ when $f'(x) = 10$?

- (A) $y = 10x - 8.05$
- (B) $y = x - 8.05$
- (C) $y = x - 3.05$
- (D) $y = 10x - 11.5$
- (E) $y = 10x - 3.05$

Answer : (E) $y=10x-3.05$

Note: we need the point to write the equation by using $f'(x) = 10$
find x then substitute to find y

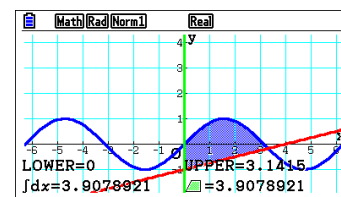
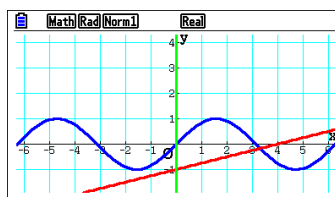
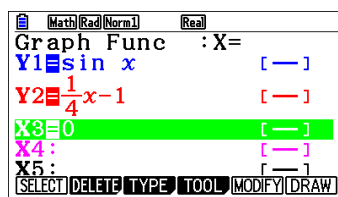
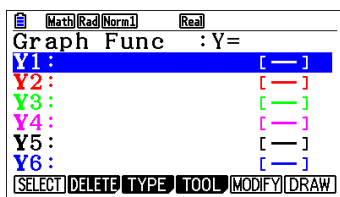


OPTN F4 F5 2 SHIFT In 2 X,θ,T ► SHIFT • 1 0) EXE EXIT
MENU 5 SHIFT In 2 X,θ,T EXE EXE F4 F2 • 8 0 4 7 2 EXE EXE

Q.5. What is the area of the region bounded by $y = \sin x$, $y = \frac{1}{4}x - 1$, and the y-axis?

- (A) 0.772
- (B) 2.815
- (C) 3.926
- (D) 5.552
- (E) 34.882

Answer : (c) 3.926



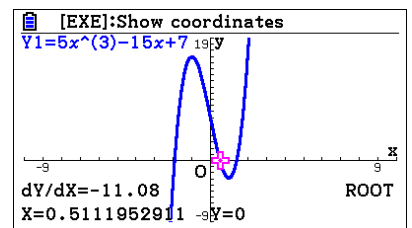
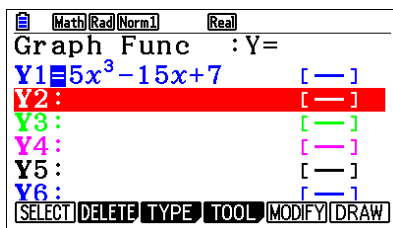
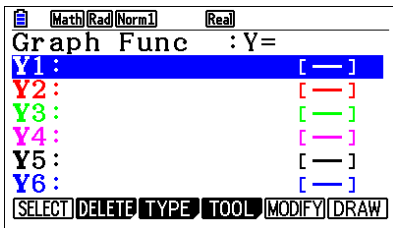
sin X,θ,T EXE 1 ▼ 4 ► X,θ,T
EXE F3 F4 0 EXE F6 F5 F6 F3 F4 ► ► EXE

Q.6. The function f whose derivative is given by $f'(x) = 5x^3 - 15x + 7$ has a local maximum at $x =$

- (A) -1.930
- (B) -1.000
- (C) 0.511
- (D) 1.000
- (E) 1.419

Answer : (B) 0.511

Note: local maximum is a point which is the function goes increasing then decreasing for $f(x)$, and the graph is positive then negative for $f'(x)$

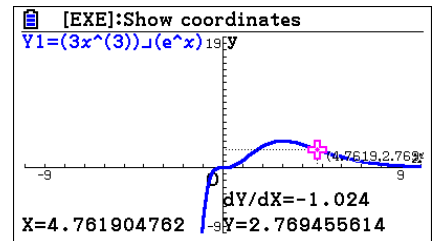
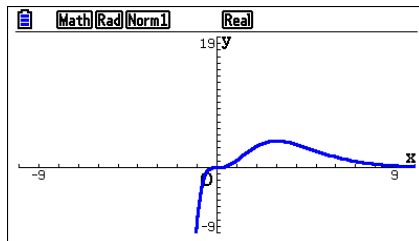
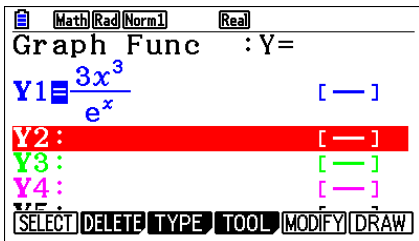


5 X,θ,T ^ 3 > = 1 5 X,θ,T + 7 EXE EXE F5 F1 >

Q.7. Let f be the function given by $f(x) = \frac{3x^3}{e^x}$. For what value of x is the slope of the line tangent to f equal to -1.024 ?

- (A) -9.004
- (B) -4.732
- (C) 1.029
- (D) 1.277
- (E) 4.797

Answer : (E) 4.797

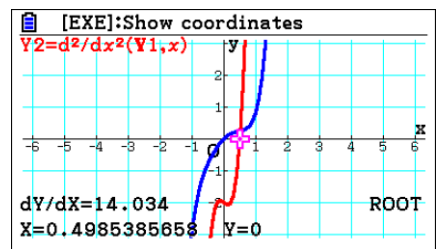
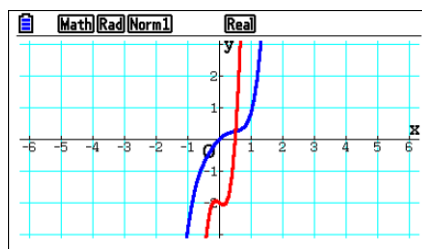
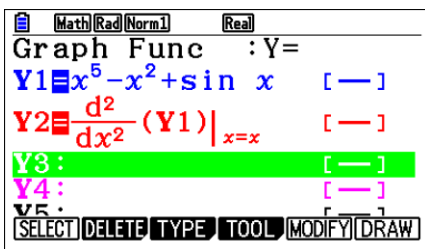


3 X,θ,T ^ 3 > = SHIFT ln X,θ,T EXE EXE F1 >

- Q.8. The graph of the function $y = x^5 - x^2 + \sin x$ changes concavity at $x =$
- (A) 0.324
 - (B) 0.499
 - (C) 0.506
 - (D) 0.611
 - (E) 0.704

Answer : (B) 0.499

Note: second derivative graph shows the concavity clearly at the root.

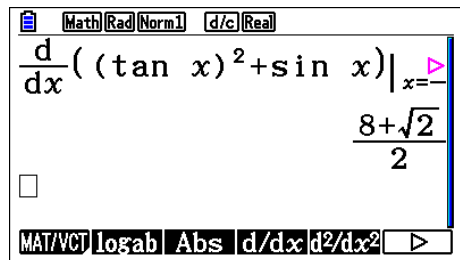
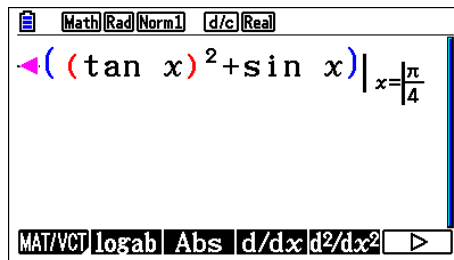


MENU 5 X,θ,T ^ 5 ► x² + sin X,θ,T EXE OPTN
 F2 F2 F1 1 ► X,θ,T EXE EXE F5 F1 ▼ EXE

- Q.9. If $f(x) = \tan^2 x + \sin x$, then $f'\left(\frac{\pi}{4}\right) =$

- (A) $\frac{4 + \sqrt{2}}{2}$
- (B) $\frac{2 + \sqrt{2}}{2}$
- (C) $\frac{8 + \sqrt{2}}{2}$
- (D) $\frac{8 - \sqrt{2}}{2}$
- (E) $\frac{4 - \sqrt{2}}{2}$

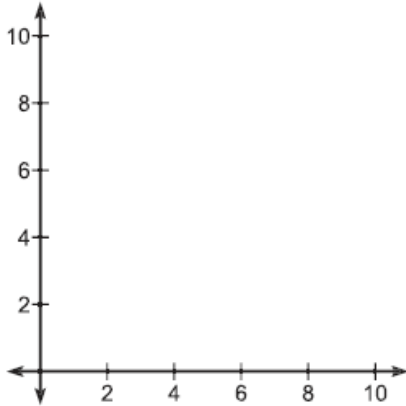
Answer : (C)



F4 F4 (tan X,θ,T) x² + sin X,θ,T ► SHIFT x10^x 4

Q.10. Let R be the region bound by $y = 2x^2 - 8x + 11$ and $y = x^2 - 4x + 10$.

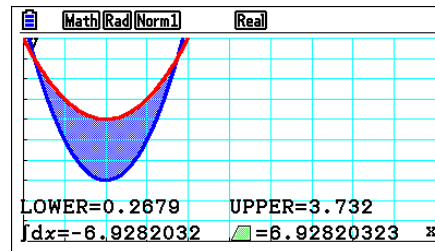
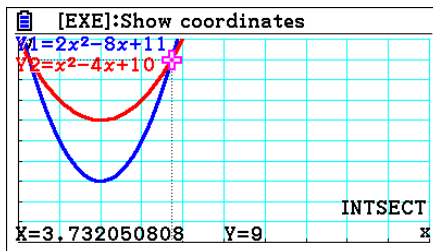
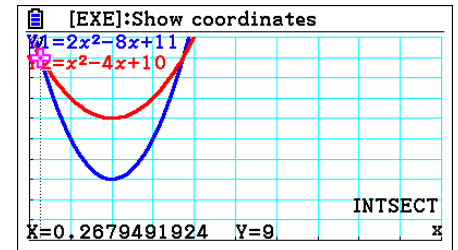
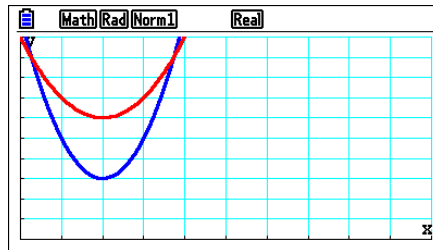
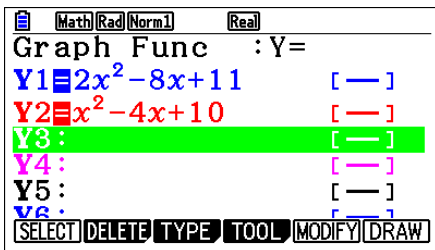
(a) Sketch the region on the axes provided.



(b) Determine the area of R .

1. Drawing the functions
2. determine the intersection points
3. Calculate the area

Note: after sketching the graph we need to determine the intersection points to find the area and volume .



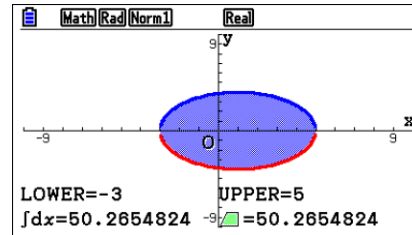
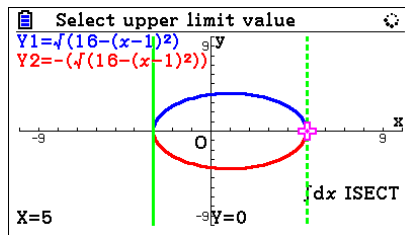
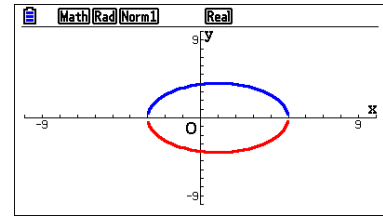
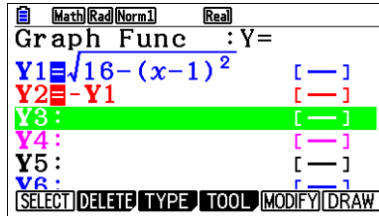
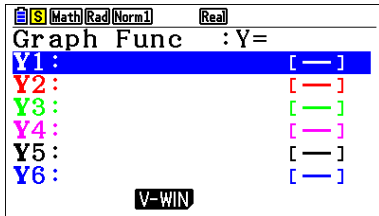
2 X,θ,T x^2 $-$ 8 X,θ,T $+$ 1 1 EXE X,θ,T x^2 $-$ 4 X,θ,T $+$ 1 0 EXE
 $F3$ 0 EXE 1 0 EXE \blacktriangledown \blacktriangledown 0 EXE 1 0 EXE EXE $F6$ $F5$ $F5$ \blacktriangleright
 $F5$ $F6$ $F3$ $F3$ EXE \blacktriangleright EXE

Answer : (6.928)

Q.11. Let f be the functions given by:

$$f(x) = \sqrt{16 - (x - 1)^2}$$

Find the area of the region enclosed by the graphs of $f(x)$ and $-f(x)$.



MENU 5 SHIFT x² 1 6 - (X,θ,T - 1) x² EXE -
 F1 1 EXE EXE F3 F3 EXIT F6 F5 F6 F3 F3 EXE ► EXE

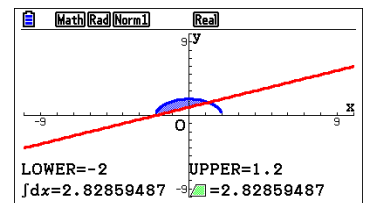
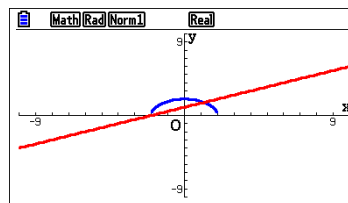
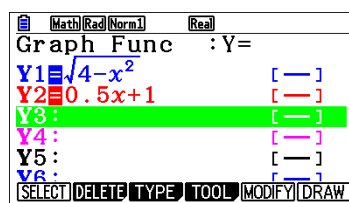
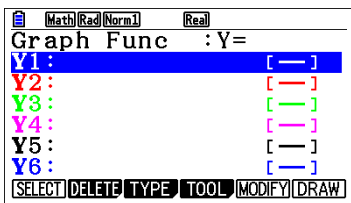
Answer : (50.265)

Q.12. Let f and g be the functions given by:

$$f(x) = \sqrt{4 - x^2}$$

$$g(x) = 0.5x + 1$$

Let R be the region in the first and second quadrants enclosed by the graphs of f and g . Find the area of R .

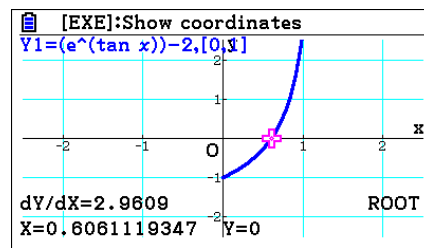
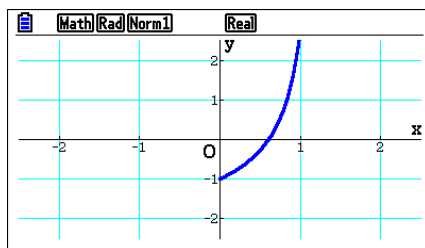
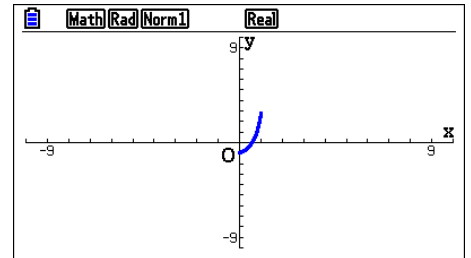
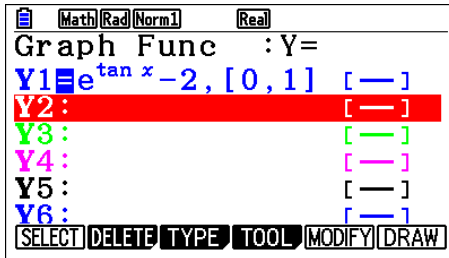
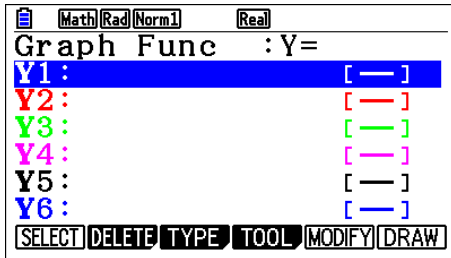


SHIFT x² 4 - X,θ,T x² EXE 0 . 5 X,θ,T + 1 EXE EXE F5 F6 F3 F3 EXE ► EXE

Answer : (2.828)

Q.13. The graph of $y = e^{\tan x} - 2$ crosses the x -axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

- (A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747



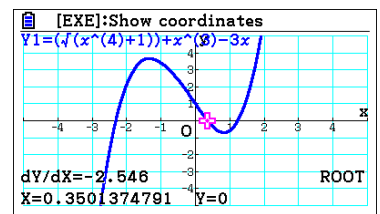
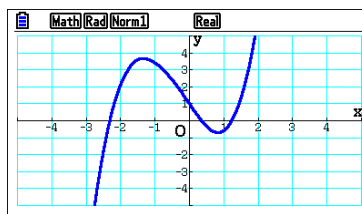
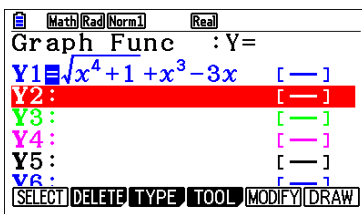
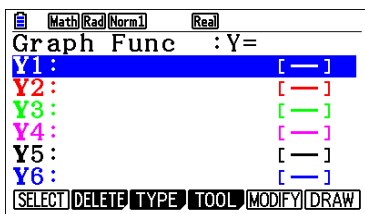
SHIFT In tan X,θ,T → - 2 , SHIFT + 0 , 1 SHIFT - EXE EXE + + F5 F1

Answer : (D) 2.961

Q.14. If $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x$, then f has a local maximum at $x =$

- (A) -2.314 (B) -1.332 (C) 0.350 (D) 0.829 (E) 1.234

Note: the maximum local are exist when the curve increasing then decreasing for $f(x)$, and if the curve above x -axis then goes down x -axis for $f'(x)$



SHIFT x^2 X,θ,T ^ 4 → + 1 → + X,θ,T ^ 3 → - 3 X,θ,T EXE EXE - F5 F1 → EXIT

Answer : (C) 0.350

- Q.15. What is the area of the region enclosed by the graphs of $y = \sqrt{4x - x^2}$ and $y = \frac{x}{2}$?
- (A) 1.707 (B) 2.829 (C) 5.389 (D) 8.886 (E) 21.447

Math Rad Norm1 Real

Graph Func : Y=

Y1 $\sqrt{4x-x^2}$ [—]

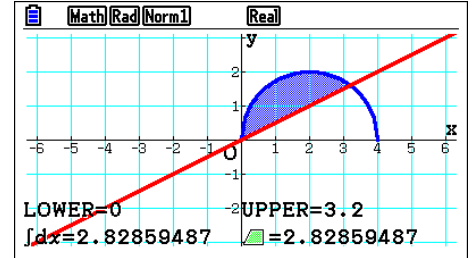
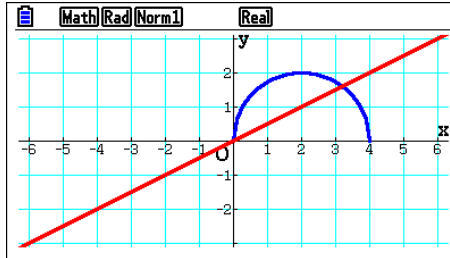
Y2 $\frac{x}{2}$ [—]

Y3 : [—]

Y4 : [—]

VF :

SELECT DELETE TYPE TOOL MODIFY DRAW



MENU 5 SHIFT x^2 4 X,θ,T x^2 EXE X,θ,T $\frac{\square}{\square}$ 2 EXE
 EXE F3 F1 EXIT F6 F5 F6 F3 F3 EXE \blacktriangleright EXE

Answer : (B) 2.829

- Q.16. A particle moves along the x -axis so that its position at time $t > 0$ is given by $x(t)$ and $\frac{dx}{dt} = -10t^4 + 9t^2 + 8t$. The acceleration of the particle is zero when $t =$
- (A) 0.387 (B) 0.831 (C) 1.243 (D) 1.647 (E) 8.094

Math Rad Norm1 Real

Graph Func : Y=

Y1 $-10x^4+9x^2+8x$ [—]

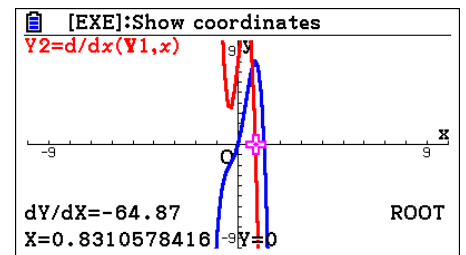
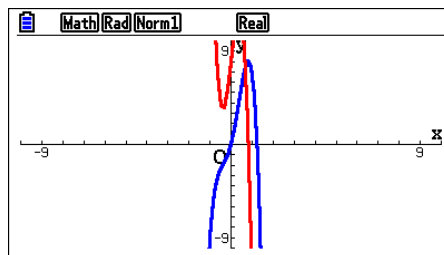
Y2 $\frac{d}{dx}(Y1)|_{x=x}$ [—]

Y3 : [—]

Y4 : [—]

Y5 :

SELECT DELETE TYPE TOOL MODIFY DRAW

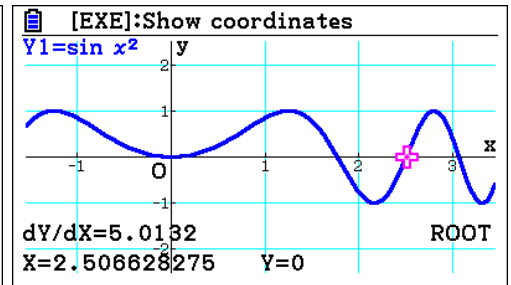
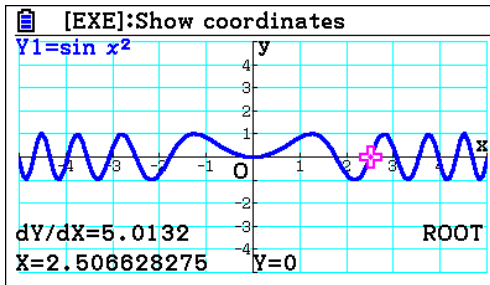
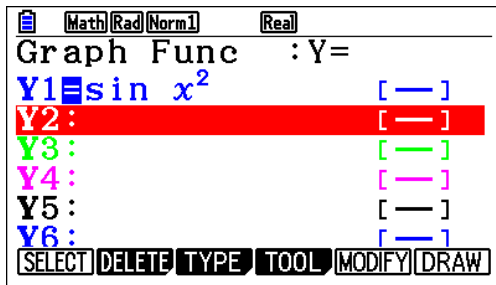


MENU 5 1 0 X,θ,T \wedge 4 \blacktriangleright + 9 X,θ,T x^2 + 8 X,θ,T
 EXE OPTN F2 F1 F1 1 \blacktriangleright X,θ,T F5 F1 \blacktriangledown EXE EXE EXE

Answer : (B) 0.831

Q.17. The first derivative of the function f is given by $f'(x) = \sin(x^2)$. At which of the following values of x does f have a local minimum?

- (A) 2.507 (B) 2.171 (C) 1.772 (D) 1.253 (E) 0



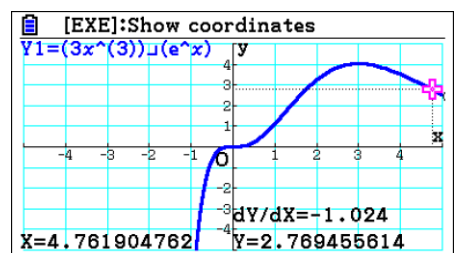
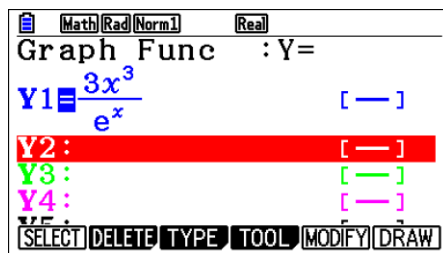
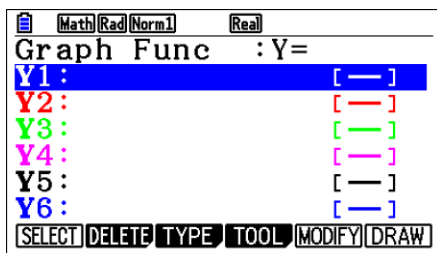
EXIT ▲ F2 F1 ▲ F2 F1 sin X,θ,T x² F5 F1 ▶▶▶

Answer : (A) 2.507

Q.18. Let f be the function given by $f(x) = \frac{3x^3}{e^x}$. For what value of x is the slope of the line tangent to f equal to -1.024 ?

- (A) -9.004
(B) -4.732
(C) 1.029
(D) 1.277
(E) 4.797

Answer : (E) 4.797



3 X,θ,T ^ 3 ▶ [] SHIFT In X,θ,T EXE EXE F1 ▶▶

Q.19. $\int_1^5 \left(\frac{3x}{x^3} \right) dx =$

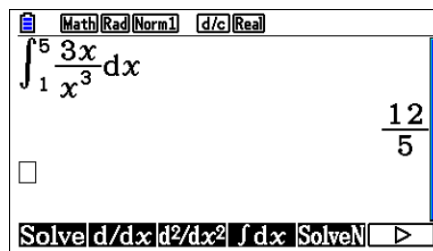
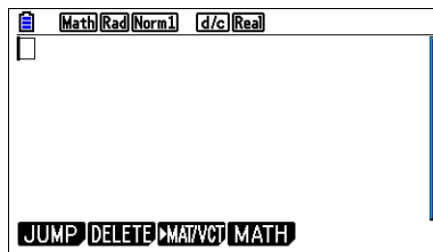
(A) $-\frac{18}{5}$

(B) $-\frac{72}{25}$

(C) $\frac{124}{125}$

(D) $\frac{126}{125}$

(E) $\frac{12}{5}$



Answer : (E) $\frac{12}{5}$

MENU 1 OPTN F4 F4 3 X,θ,T $\frac{\square}{\square}$ X,θ,T \wedge 3 \blacktriangleright \blacktriangleright \blacktriangleright 1 \blacktriangle 5 EXE

Q.20. The graph of $y = 3x^3 - 2x^2 + 6x - 2$ is decreasing for which interval(s)?

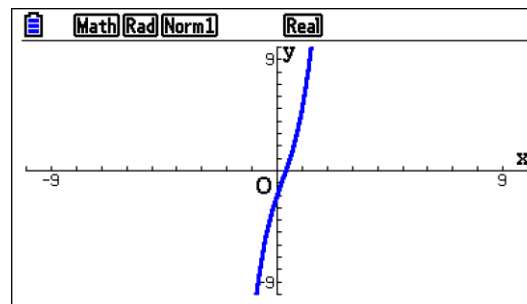
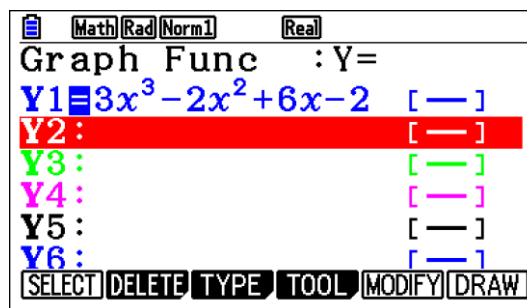
(A) $\left(-\infty, \frac{2}{9}\right)$

(B) $\left(\frac{2}{9}, \infty\right)$

(C) $\left[0, \frac{2}{9}\right]$

(D) $(-\infty, \infty)$

(E) None of the above

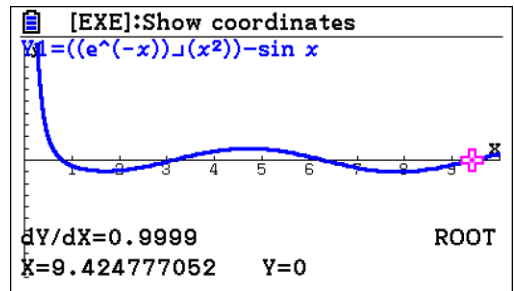
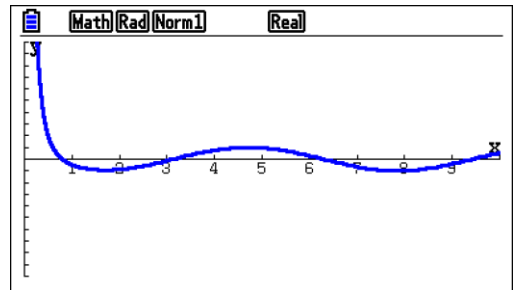
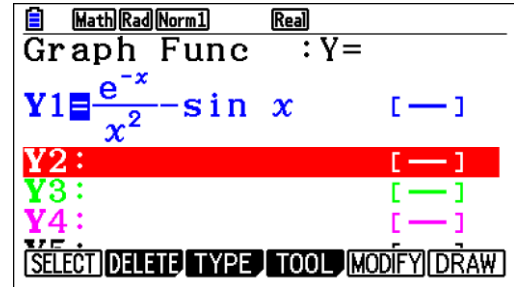


MENU 5 3 X,θ,T \wedge 3 \blacktriangleright \square 2 X,θ,T x^2 \square 6 X,θ,T \square 2 EXE EXE

Answer : (E) Non of the above

Q.21. The first derivative of a function, f , is given by $f'(x) = \frac{e^{-x}}{x^2} - \sin x$. How many critical values does f have on the open interval $(0,10)$?

- (A) One
- (B) Two
- (C) Three
- (D) Four
- (E) Five



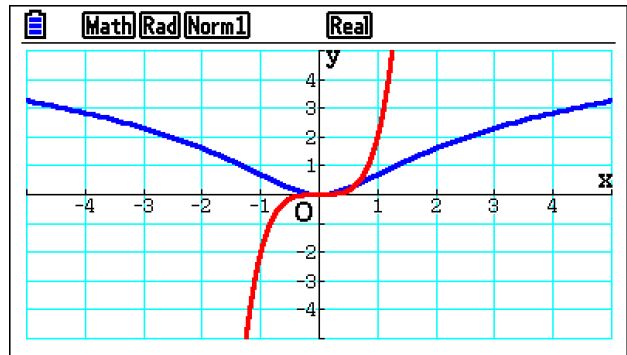
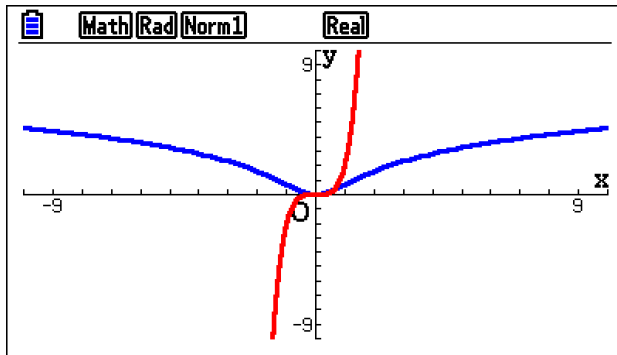
MENU 5 SHIFT ln - X,θ,T > X,θ,T x² > -
 sin X,θ,T EXE SHIFT F3 0 EXE 1 0 EXE EXE F6 F5 F1 > > >

Answer : (D) Four (intersection with x-axis)

Q.22. Let f be the function defined by $f(x) = \ln(x^2 + 1)$, and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at $x = 2$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

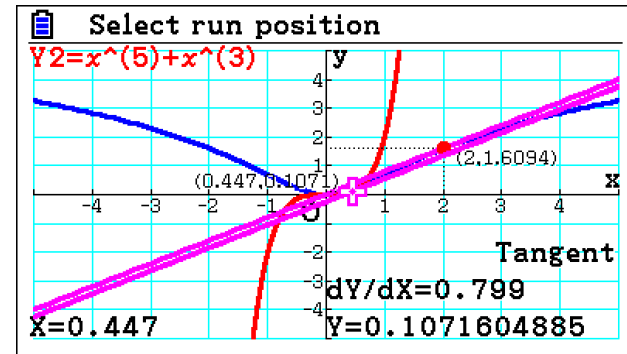
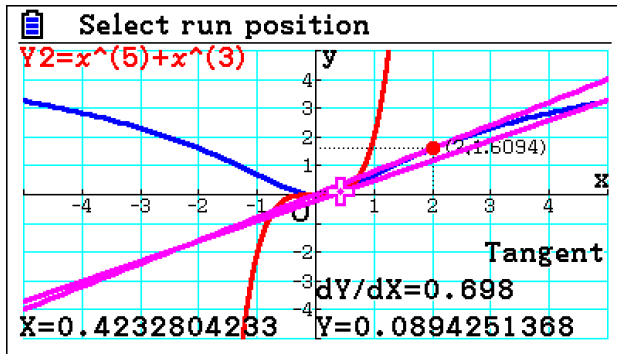
- (A) 0.246 (B) 0.430 (C) 0.447 (D) 0.790

Graph the two functions then for $f(x)$ sketch the tangent line after that you can sketch parallel tangent line for $g(x)$.



MENU 5 ln (X,θ,T x² + 1) EXE
 X,θ,T ^ 5 ► + X,θ,T ^ 3 EXE EXE

F3 - 5 EXE 5 EXE ▼ ▼ -
 5 EXE 5 EXE EXE EXE



F4 F2 2 EXE EXE ▼ ◀ ◀ ◀ ◀
 ◀ ◀ ◀ ◀ ◀ ◀ ◀ ◀ ◀ ◀
 ◀ ◀ ◀ ◀ ◀ ◀ ◀ ◀ ◀ ◀

◻ 4 4 7 EXE EXE

Answer : (C) 0.447

Q.23. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \leq t \leq 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

- How many people enter the line for the escalator during the time interval $0 \leq t \leq 300$?
- During the time interval $0 \leq t \leq 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?
- For $t > 300$, what is the first time t that there are no people in line for the escalator?
- For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

(a) During the interval the number of people are the integration of rate.

Math (Rad) (Norm) (d/c) (Real)

$$\int_0^{300} 44 \left(\frac{x}{100} \right)^3 \left(1 - \frac{x}{300} \right)^7 dx = 270$$

Solve d/dx d²/dx² ∫ dx SolveN ▶

Answer : 270 people

MENU 1 OPTN F4 F4 4 4 (X,θ,T) 1 0 0) ^ 3) 1 - X,θ,T 3 0 0) ^ 7) 0 3 0 0 EXE

(b) The function $r(t)$ is rate in and 0.7 us rate out and at $t=0$ there are 20 people .

Math (Rad) (Norm) (d/c) (Real)

$$20 + \int_0^{300} 44 \left(\frac{x}{100} \right)^3 \left(1 - \frac{x}{300} \right)^7 dx = 80$$

Solve d/dx d²/dx² ∫ dx SolveN ▶

Answer : 80 people

OPTN F4 2 0 + F4 4 4 (X,θ,T) 1 0 0) ^ 3) 1 - X,θ,T 3 0 0) ^ 7) - 0 7) 0 3 0 0 EXE

(c) At $t=300$ there are 80 people and for $t > 300$ the $r(t) = 0$ but the rate out $r(t) = 0.7$, we need first t with no people

Calculator screen showing the equation $20 + \int_0^x 44 \left(\frac{x}{100} \right) \left(1 - \frac{x}{300} \right) dx = 80$ and the solution $\text{SolveN} \left(80 + \int_{300}^x (0 - 0.7) dx \right) \{ 414.2857143 \}$.

Answer : $t = 414.3$ sec

F5 8 0 + F4 (0 - . 7) > 3
0 0 < X,θ,T > SHIFT . 0) EXE EXIT S+D

(d) Number of people is minimum so we need to find the critical points ($r(t) - 0.7 = 0$) and make a table to compare the critical points.

Calculator screen showing the solution for the minimum number of people: $\text{SolveN} \left(80 + \int_{300}^x (0 - 0.7) dx \right) \{ 414.2857143 \}$ and $\text{SolveN} \left(44 \left(\frac{x}{100} \right)^3 \left(1 - \frac{x}{300} \right) \right) \{ 33.01329783, 166.574 \}$.

x	Y2	Y'2
0	20	-0.7
33.013	3.8034	-1E-5
166.57	158.07	1.8E-5
300	80	-0.7

F5 4 4 (X,θ,T > 1 0 0 >) ^ 3 > (1 -
X,θ,T > 3 0 0 >) ^ 7 > - 0 . 7 SHIFT . 0) EXE EXIT

MENU 7 4 4 (X,θ,T > 1 0 0 >) ^ 3 > (1 -
7 > - . 7 EXE 2 0 + OPTN F2 F3 VARS F4 F1 1 > 0 < X,θ,T EXE < < F1
F6 0 EXE > 3 3 . 0 1 3 EXE > 1 6 6 . 5 7 4 EXE > 3 0 0 EXE

Answer : Minimum people at $t = 33$ & maximum at $t = 166.5$

Q.24. A particle moves along the x -axis with velocity given by $v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \leq t \leq 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

- (a) Find the acceleration of the particle at time $t = 3$.
- (b) Find the position of the particle at time $t = 3$.
- (c) Evaluate $\int_0^{3.5} v(t) dt$, and evaluate $\int_0^{3.5} |v(t)| dt$. Interpret the meaning of each integral in the context of the problem.
- (d) A second particle moves along the x -axis with position given by $x_2(t) = t^2 - t$ for $0 \leq t \leq 3.5$. At what time t are the two particles moving with the same velocity?

(a) Acceleration $a(t) = v'(t)$

Calculator screen showing the derivative of the velocity function at $t=3$. The display shows: $\frac{d}{dx} \left(\frac{10 \sin 0.4x^2}{x^2 - x + 3} \right) \Big|_{x=3}$ and the result -2.118195009 . The bottom menu bar includes options: Solve, d/dx, d²/dx², ∫ dx, SolveN, and a right arrow.

Answer $a = -2.118$

Calculator keypad showing the sequence of keys used to calculate the derivative: MENU, 1, OPTN, F4, F2, 1, 0, sin, 0, ., 4, X,θ,T, x², X,θ,T, x², -, X,θ,T, +, 3, >, >, 3, EXE.

(b) Position $x = -5$ at $t=0$, $x(t) = \int v(t) dt$

Calculator screen showing the integral of the velocity function from $t=0$ to $t=3$. The display shows: $\int dx \left(\frac{10 \sin 0.4x^2}{x^2 - x + 3} \right) \Big|_{x=3}$ and the result -2.118195009 . Below this, it shows $-5 + \int_0^3 \frac{10 \sin 0.4x^2}{x^2 - x + 3} dx$ and the result -1.760213187 . The bottom menu bar includes options: Solve, d/dx, d²/dx², ∫ dx, SolveN, and a right arrow.

Answer $x = -1.76$

Calculator keypad showing the sequence of keys used to calculate the integral: -, 5, +, OPTN, F4, F4, 1, 0, sin, 0, ., 4, X,θ,T, x², X,θ,T, x², -, X,θ,T, +, 3, >, >, 0, <, 3, EXE.

(c) Evaluations , the particle displacement from 0 to 3.5 is 2.844 , and the particle distance traveled is 3.737

Math Rad Norm1 d/c Real

$$\int_0^{3.5} \frac{10 \sin 0.4x^2}{x^2 - x + 3} dx$$

2.843944475

Solve d/dx d²/dx² ∫ dx SolveN ▶

Answer: 2.844

Math Rad Norm1 d/c Real

$$\int_0^{3.5} \left| \frac{10 \sin 0.4x^2}{x^2 - x + 3} \right| dx$$

3.737079816

Abs Int Frac Rnd Intg ▶

Answer : 3.737

F4 $\frac{\square}{\square}$ 1 0 sin 0 \cdot 4 X,θ,T x² ▼ X,θ,T x² -

X,θ,T + 3 ▶ ▶ 0 ▲ 3 \cdot 5 EXE F4 OPTN F6 F4 F1 $\frac{\square}{\square}$ 1 0 sin 0

\cdot 4 X,θ,T x² ▼ X,θ,T x² - X,θ,T + 3 ▶ ▶ ▶ 0 ▲ 3 \cdot 5 EXE

(d) $V_1(t) = V_2(t)$, $V_2(t)=2t-1$

Math Rad Norm1 d/c Real

$$\int_0^{3.5} \left| \frac{10 \sin 0.4x^2}{x^2 - x + 3} \right| dx$$

3.737079816

SolveN $\left(2x - 1 = \frac{10 \sin 0.4x^2}{x^2 - x + 3} \right)$ ▶

{1.570540044}

Solve d/dx d²/dx² ∫ dx SolveN ▶

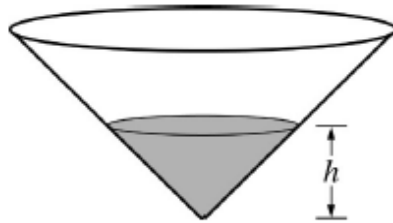
Answer : t = 1.57

OPTN F4 F5 2 X,θ,T SHIFT \cdot $\frac{\square}{\square}$ 1 0 sin 0 \cdot 4

X,θ,T x² ▼ X,θ,T x² - X,θ,T + 3 ▶) EXE EXIT

Q.25. Free Response: Section II, Part A

A graphing calculator is required for problems on this part of the exam.



The height of the water in a conical storage tank, shown above, is modeled by a differentiable function h , where $h(t)$ is measured in meters and t is measured in hours. At time $t = 0$, the height of the water in the tank is 25 meters. The height is changing at the rate

$$h'(t) = 2 - \frac{24e^{-0.025t}}{t+4} \text{ meters per hour for } 0 \leq t \leq 24.$$

- (a) When the height of the water in the tank is h meters, the volume of water is $V = \frac{1}{3}\pi h^3$. At what rate is the volume of water changing at time $t = 0$? Indicate units of measure.
- (b) What is the minimum height of the water during the time period $0 \leq t \leq 24$? Justify your answer.

(a) At $t=0$, $h=25$

$$\frac{dv}{dt} = \frac{1}{3} \times \pi \times 3h^2 \frac{dh}{dt}$$

Answer : $\frac{dv}{dt} = -2500\pi$

- (b) The absolute minimum must be a critical point for the endpoint.
So we need to solve $h'(t)=0$ to get $t = 6.26$,
and at $t=0$ $h=25$. the minimum is 25 + the integration from 0 to 6.25

Math Rad Norm1 d/c Real

$$\frac{2}{3}\pi \times 3 \times (25^4) \times \left(2 - \frac{24e^{-0.025x}}{x+4} \right)$$

$$-2500\pi$$

SolveN $\left(2 - \frac{24e^{-0.025x}}{x+4} = 0 \right)$

{ 6.261256003 }

Solve d/dx d²/dx² ∫ dx SolveN ▶

OPTN F4 F5 2 = 2 4 SHIFT ln =

0 . 0 2 5 X,θ,T ▼ X,θ,T + 4 ▶ SHIFT . 0) EXE EXIT

Math Rad Norm1 d/c Real

SolveN $\left(2 - \frac{24e^{-0.025x}}{x+4} = 0 \right)$

{ 6.261256003 }

$$25 + \int_0^{6.26} 2 - \frac{24e^{-0.025x}}{x+4} dx$$

16.33873178

Solve d/dx d²/dx² ∫ dx SolveN ▶

Answer : h = 16.34

2 5 + F4 2 = 2 4 SHIFT ln =

0 . 0 2 5 X,θ,T ▼ X,θ,T + 4 ▶ ▶ 0 ▲ 6 . 2 6 EXE