Exploring the horizontal shift of a parabola

Stephen Sproule
St John's College, Johannesburg, South Africa

Introduction

Advancements in scientific calculators are substantially contributing to the teaching of mathematics in developing nations. Computers and even graphics calculators are cost prohibitive for the vast majority of students in many developing countries. However with progress in scientific calculators, such as the Casio ES series (Casio FX-82ES ; Casio FX-82ES – PLUS ; Casio FX-991ES ; Casio FX-991ES – PLUS), mathematics teachers are able to use this technology as a teaching tool not just a calculating device.

Teacher's notes

This lesson is an investigation of the horizontal shift of the parabolic function, $f(x) = (x - p)^2$.
It is part of a set of investigations, exploring the horizontal and vertical shifts of the parabola, rectangular hyperbola and exponential function.

Pre-knowledge: Students must have drawn a few graphs of the parabola in the form, $f(x) = x^2$. Students do not need to be able to use the properties of the parabola to draw the graph. This activity will use the table method to draw the graph.

Calculator's role: Requiring students to repeatedly complete the table for a function can be cumbersome and requiring it for a number of shifts of a function would waste valuable teaching time. The calculator helps make the activity time efficient and limits computation errors, ensuring greater access to the actual exploration.

Mathematics: In this activity we use the notation $(y - q) = x^2$ instead of $f(x) = x^2 + q$. This ensures that students associate the shifts with $(y - q)$ and $(x - p)$. The teacher can then later introduce $f(x) = x^2 + q$ as common practice.

Equipment: 10cm × 10cm piece of tracing (patty) paper

Grade: 10 or 11

Lesson

- The worksheets given below come in a set of four. Each worksheet looks remarkably similar but you will notice the function to be drawn is different on each worksheet.
- Divide your class into groups of 3 or 4 students.
- Give each student in a group a separate sheet from the set and a piece of tracing paper. That is, each student in the group must be drawing a different shift of the hyperbola.
- Students complete steps 1-8 individually. Here they use the calculator's **TABLE** mode. Although most students learn how to use this quickly, instructions for entering functions in the table mode are given below. I recommend that each group member draws his or her assigned graph in a different colour pen or pencil, and then labels the graph with the equation of the function.
- I usually stop the class at this point and ask all the students to mentally anticipate what the change in the equation might be doing to the graph.
- Groups now complete steps 9-11 together. During the group discussion they are focused on comparing the transformation of the graphs and then what mathematical details in the equation of the function produce that transformation.
- The conclusions (steps 12-15) may be completed individually, in groups or through a whole class discussion.

Further class discussion

a) Discuss the fact that the graph of the function exists for values not in the table, both for Real numbers between the given Integers and for numbers to infinity.

b) What might the function look like to produce a vertical shift?

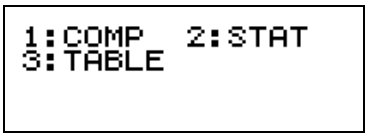
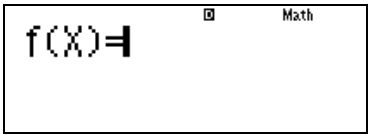
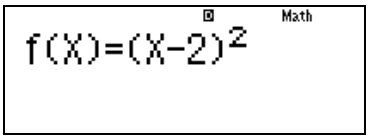
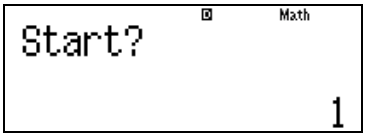
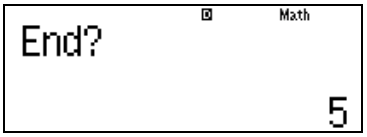
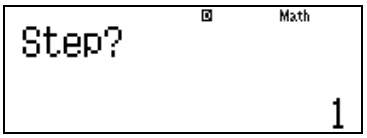
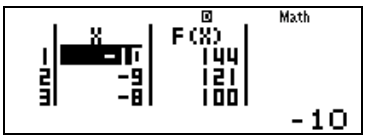
c) What table of values would you give someone to help them draw $f(x) = (x + 11)^2$?

d) How could we draw a sketch graph of a 'shifted' parabola without using a table of values?

Conclusion

In a computer based investigation of graph transformations students tend to see the graph's 'movement but need specific teacher direction to pay sufficient attention to the mathematical details of the transformation. For example, the students see the horizontal shift but don't identify the difference between $y = (x - 3)^2$ and $y = (x + 3)^2$. Students comment that they learned more by exploring the ideas with a Casio calculator, tracing paper and directed group discussion.

Entering a parabolic function on the Casio *fx*82ES or Casio *fx*82ES-PLUS

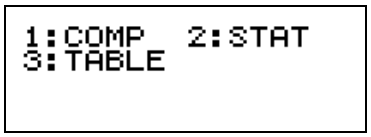
Action	Buttons to press	Resulting screen
Setting the mode	MODE	
	3	
Entering the function	(ALPHA) − 2) x^2	
	=	
Setting the domain	(−) 1 0 =	
	1 0 =	
	1 =	

Exploring $y = f(x) = (x-p)^2$ – person 1

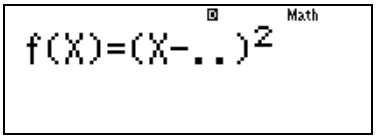
We are going to explore the influence of parameter ***p*** on the graph of the function $f(x) = (x-p)^2$

Steps

1. The graph of $f(x) = x^2$ has been drawn on the axes below.
2. Look at the graph given on the axes and suggest what the graph of $f(x) = (x-4)^2$ might look like?
3. On your Casio *fx*82ES (PLUS) calculator, press MODE and the follow the instructions:



Press 3



(X is ALPHA))

4. Complete the table for $f(x) = (x-4)^2$ using your calculator:

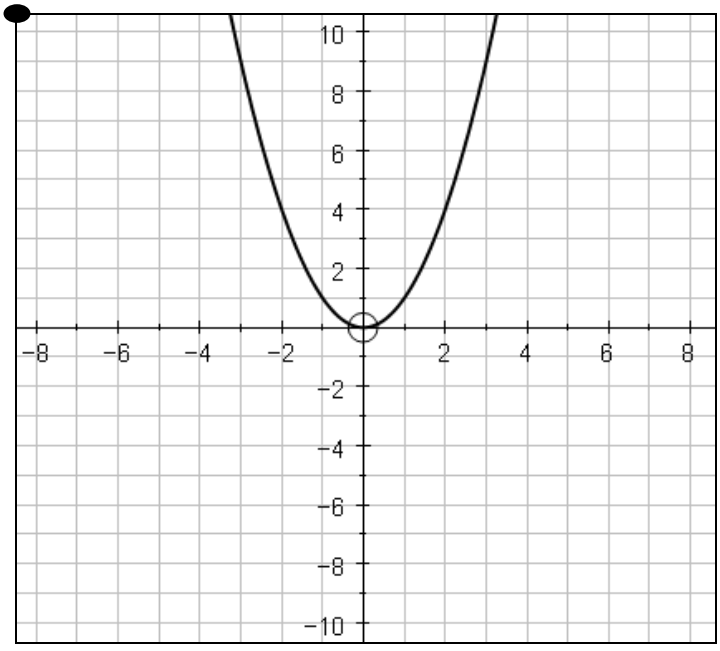
x	**Start?** 1	2	3	4	5	6	**End?** 7
$f(x) = (x-4)^2$							

5. Explain why the values of $f(x)$ are the same on both sides of 4: . . . . . .

. . . . . .

6. Lay the tracing paper over the axes.
7. Plot the points (x, y) from the table on the tracing paper and draw a sketch graph of $f(x) = (x-4)^2$.

Place the corner of your tracing paper here



8. Compare your graph to the given graph? (What is the same? What is different?)

. . . . . .

. . . . . .

. . . . . .

Group discussion

Collect all the graphs from your group. Lay them on top of each other on the axes (remembering to place the top left corner on the dot). Use the multiple graphs to answer the following questions.

9. Compare the set of graphs to the given graph?

. . . . . .

. . . . . .

. . . . . .

10. Give the co-ordinates of the minimum point on the graph: . . . . . .

11. Compare the table given to you with the tables of the other members of your group. Why were you each given a different table? . . . . . .

. . . . . .

Conclusions

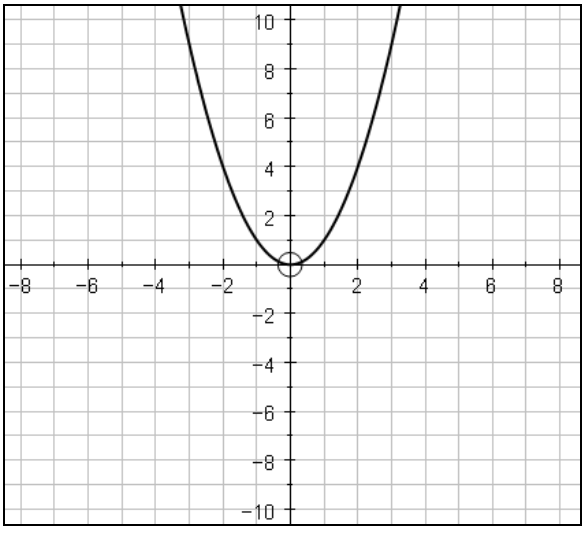
12. Is there a vertical or horizontal asymptote of the graph? . . . . . .

13 Give the co-ordinates of the minimum point of $f(x) = (x - p)^2$: . . . . . .

16. In the graph $f(x) = (x - p)^2$ the parameter, ***p***, alters the graph by: . . . . . .

. . . . . .

17. Draw an estimate of what you think the graph of $y - 3 = (x - 4)^2$ might look like?

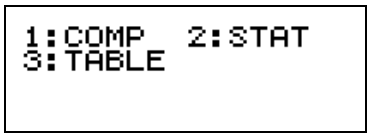


Exploring $y = f(x) = (x-p)^2$ – person 2

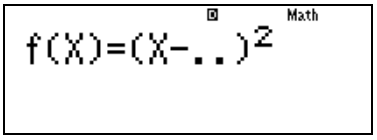
We are going to explore the influence of parameter ***p*** on the graph of the function $f(x) = (x-p)^2$

Steps

1. The graph of $f(x) = x^2$ has been drawn on the axes below.
2. Look at the graph given on the axes and suggest what the graph of $f(x) = (x-2)^2$ might look like?
3. On your Casio *fx*82E (PLUS) calculator, press [MODE] and the follow the instructions:



Press [3]



(X is [ALPHA] [)])

4. Complete the table for $f(x) = (x-2)^2$ using your calculator:

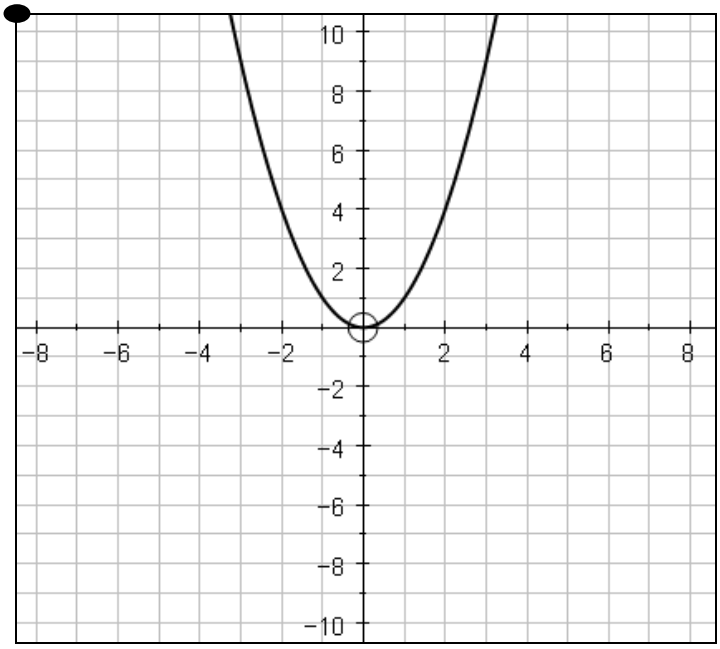
x	**Start?** -1	0	1	2	3	4	**End?** 5
$f(x) = (x-2)^2$							

5. Explain why the values of $f(x)$ are the same on both sides of 2: . . . . . .

. . . . . .

6. Lay the tracing paper over the axes.
7. Plot the points (x, y) from the table on the tracing paper and draw a sketch graph of $f(x) = (x-2)^2$.

Place the corner of your tracing paper here



8. Compare your graph to the given graph? (What is the same? What is different?)

. . . . . .

. . . . . .

. . . . . .

Group discussion

Collect all the graphs from your group. Lay them on top of each other on the axes (remembering to place the top left corner on the dot). Use the multiple graphs to answer the following questions.

9. Compare the set of graphs to the given graph?

. . . . . .

. . . . . .

. . . . . .

10. Give the co-ordinates of the minimum point on the graph: . . . . . .

11. Compare the table given to you with the tables of the other members of your group. Why were you each given a different table? . . . . . .

. . . . . .

Conclusions

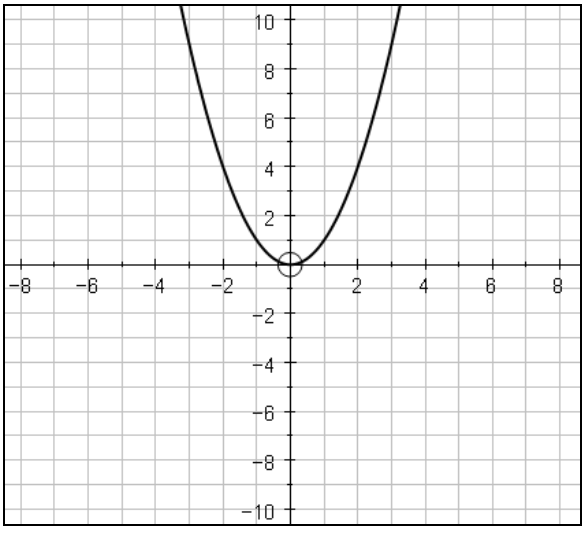
12. Is there a vertical or horizontal asymptote of the graph? . . . . . .

13 Give the co-ordinates of the minimum point of $f(x) = (x - p)^2$: . . . . . .

16. In the graph $f(x) = (x - p)^2$ the parameter, ***p***, alters the graph by: . . . . . .

. . . . . .

17. Draw an estimate of what you think the graph of $y - 3 = (x - 2)^2$ might look like?

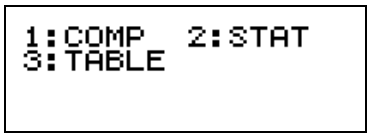


Exploring $y = f(x) = (x-p)^2$ – person 3

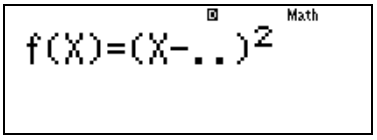
We are going to explore the influence of parameter ***p*** on the graph of the function $f(x) = (x-p)^2$

Steps

1. The graph of $f(x) = x^2$ has been drawn on the axes below.
2. Look at the graph given on the axes and suggest what the graph of $f(x) = (x+2)^2$ might look like?
3. On your Casio *fx*82ES (PLUS) calculator, press MODE and the follow the instructions:



Press 3



(X is ALPHA))

4. Complete the table for $f(x) = (x+2)^2$ using your calculator:

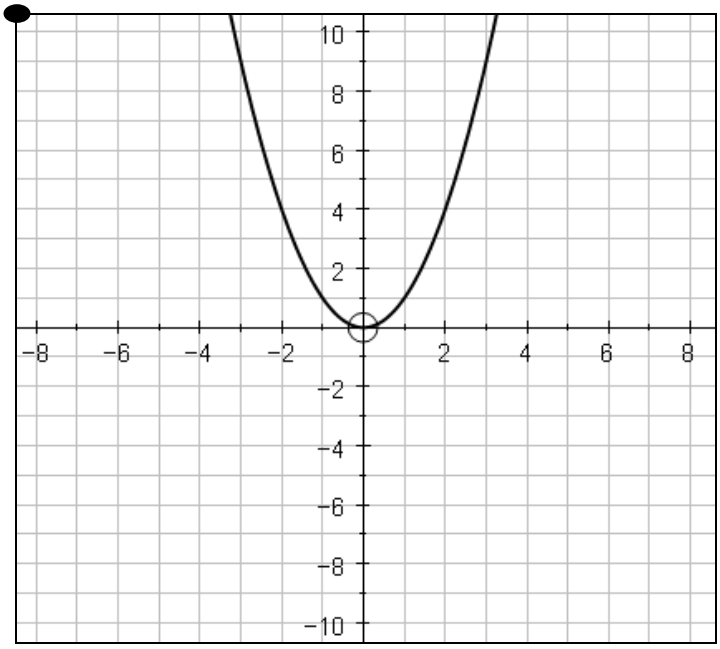
x	**Start?** -5	-4	-3	-2	-1	0	**End?** 1
$f(x) = (x+2)^2$							

5. Explain why the values of $f(x)$ are the same on both sides of -2: . . . . . .

. . . . . .

6. Lay the tracing paper over the axes.
7. Plot the points (x, y) from the table on the tracing paper and draw a sketch graph of $f(x) = (x+2)^2$.

Place the corner of your tracing paper here



8. Compare your graph to the given graph? (What is the same? What is different?)

. . . . . .

. . . . . .

. . . . . .

Group discussion

Collect all the graphs from your group. Lay them on top of each other on the axes (remembering to place the top left corner on the dot). Use the multiple graphs to answer the following questions.

9. Compare the set of graphs to the given graph?

. . . . . .

. . . . . .

. . . . . .

10. Give the co-ordinates of the minimum point on the graph: . . . . . .

11. Compare the table given to you with the tables of the other members of your group. Why were you each given a different table? . . . . . .

. . . . . .

Conclusions

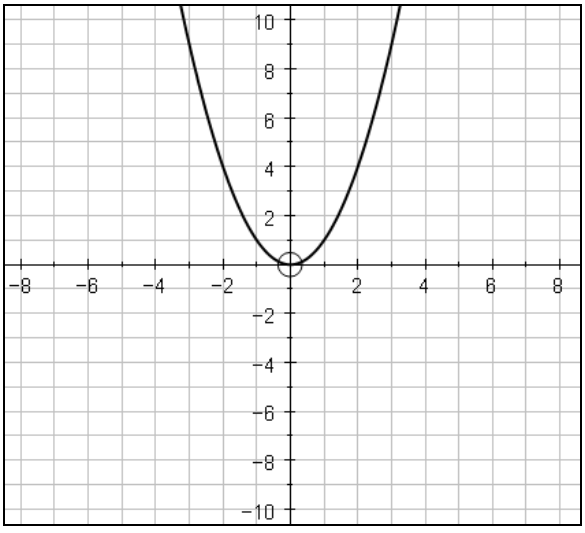
12. Is there a vertical or horizontal asymptote of the graph? . . . . . .

13 Give the co-ordinates of the minimum point of $f(x) = (x - p)^2$: . . . . . .

16. In the graph $f(x) = (x - p)^2$ the parameter, ***p***, alters the graph by: . . . . . .

. . . . . .

17. Draw an estimate of what you think the graph of $y - 3 = (x + 2)^2$ might look like?

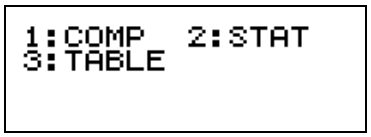


Exploring $y = f(x) = (x-p)^2$ – person 4

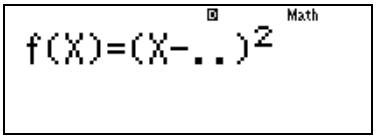
We are going to explore the influence of parameter ***p*** on the graph of the function $f(x) = (x-p)^2$

Steps

1. The graph of $f(x) = x^2$ has been drawn on the axes below.
2. Look at the graph given on the axes and suggest what the graph of $f(x) = (x+5)^2$ might look like?
3. On your Casio *fx*82ES (PLUS) calculator, press MODE and the follow the instructions:



Press 3



(X is ALPHA))

4. Complete the table for $f(x) = (x+5)^2$ using your calculator:

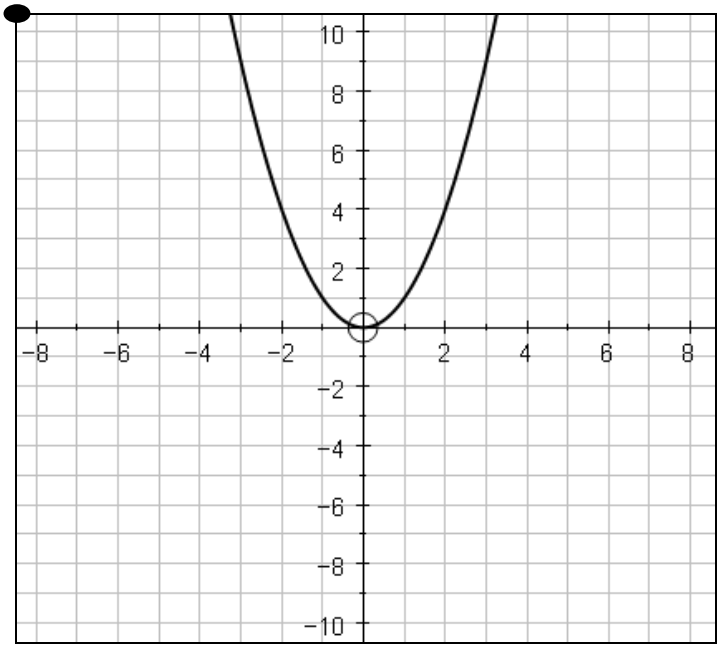
x	**Start?** -8	-7	-6	-5	-4	-3	**End?** -2
$f(x) = (x+5)^2$							

5. Explain why the values of $f(x)$ are the same on both sides of -5: . . . . . .

. . . . . .

6. Lay the tracing paper over the axes.
7. Plot the points (x, y) from the table on the tracing paper and draw a sketch graph of $f(x) = (x+5)^2$.

Place the corner of your tracing paper here



8. Compare your graph to the given graph? (What is the same? What is different?)

. . . . . .

. . . . . .

. . . . . .

Group discussion

Collect all the graphs from your group. Lay them on top of each other on the axes (remembering to place the top left corner on the dot). Use the multiple graphs to answer the following questions.

9. Compare the set of graphs to the given graph?

. . . . . .

. . . . . .

. . . . . .

10. Give the co-ordinates of the minimum point on the graph: . . . . . .

11. Compare the table given to you with the tables of the other members of your group. Why were you each given a different table? . . . . . .

. . . . . .

Conclusions

12. Is there a vertical or horizontal asymptote of the graph? . . . . . .

13 Give the co-ordinates of the minimum point of $f(x) = (x - p)^2$: . . . . . .

16. In the graph $f(x) = (x - p)^2$ the parameter, ***p***, alters the graph by: . . . . . .

. . . . . .

17. Draw an estimate of what you think the graph of $y - 3 = (x + 5)^2$ might look like?